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A CAUCHY PROBLEM FOR ELLIPTIC INVARIANT DIFFERENTIAL OPERATORS AND CONTINUITY OF A GENERALIZED BEREZIN TRANSFORM

by Bent ØRSTED & Jorge VARGAS (*)

Abstract. — In this note, we generalize the results in our previous paper on the Casimir operator and Berezin transform, by showing the (L^2, L^2)-continuity of a generalized Berezin transform associated with a branching problem for a class of unitary representations defined by invariant elliptic operators; we also show, that under suitable general conditions, this generalized Berezin transform is (L^p, L^p)-continuous for $1 \leqslant p \leqslant \infty$.

Résumé. — Dans cette note, nous généralisons les résultats de notre article précédent sur l'opérateur de Casimir et sur la transformée de Berezin, en prouvant la continuité (L^2, L^2) d'une transformée de Berezin généralisée associée à un problème de bifurcation pour une classe de représentations unitaires définie par des opérateurs elliptiques invariants. Nous prouvons aussi que, sous des conditions générales adéquates, cette transformée de Berezin généralisée est (L^p, L^p)-continue pour $1 \leqslant p \leqslant \infty$.

1. Introduction

A basic problem in representation theory of Lie groups is to derive "branching laws". By this we mean, for a given unitary irreducible representation of an ambient group G, consider its restriction to a fixed subgroup H and find the decomposition as a direct integral, and in particular compute the multiplicity of each irreducible factor of the restriction. There is a vast literature on this subject, and here we just direct the reader's attention to the extensive reviews of [6], [7] and references therein. One way

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to attack this problem is to consider geometric realizations of the representation in question. For example, we might consider unitary representations realized as the L^2-kernel of an invariant differential operator acting on sections of a homogeneous vector bundle and study normal derivatives along a submanifold. This path has been followed in [9], and in [4] for the case of holomorphic vector bundles, and this idea occurs in many other works. A closely connected problem is that of studying certain Cauchy problems for invariant elliptic operators.

The object of this note is to show some perhaps surprising facts for these Cauchy problems and the corresponding branching problems. Our methods are very similar to those in our previous paper [9]; but in the present paper we show that the explicit construction of Hotta can be replaced by a general L^2 - kernel of an invariant elliptic operator, thus simplifying and generalizing the argument.

In order to state the main result, we consider a connected semisimple matrix Lie group G. Henceforth, we fix a connected reductive subgroup H of G and a maximal compact subgroup K of G such that $H \cap K$ is a maximal compact subgroup of H. We fix Haar measures in G and H and assume that the group G has a nonempty Discrete Series. For general facts and notation, see for example [5]. Let (τ, W) be a finite dimensional representation of K. Let $E := G \times_\tau W \longrightarrow G/K$ be the $G-$homogeneous, Hermitian, smooth vector bundle attached to the representation τ. We denote its space of L^2- (resp. smooth) sections by

$$L^2(G,\tau) = \left\{ f : G \longrightarrow W : f(gk) = \tau(k^{-1}) f(g),\, g \in G,\, k \in K, \right.$$
$$\left. \int_G |f(g)|^2 dg < \infty \right\}$$

(resp. $C^\infty(G,\tau)$). The Lie algebra of a Lie group will be denoted by the corresponding German lower case letter, the complexification of a real Lie algebra will be denoted by adding the subscript $\mathbb{C}$. For this note,

D is a $G-$homogeneous, elliptic differential operator on $C^\infty(G,\tau)$.

In [1] we find a proof of the fact that the minimal extension of D agrees with the maximal extension. Therefore, the L^2-kernel of D is a well defined object. For this, we consider the closure $\bar{D}$ of D as an unbounded linear operator on $L^2(G,\tau)$ and then

$$L^2(KerD) := \{ f \in L^2(G,\tau) : \;\; \bar{D}(f) = 0 \}$$

is a well defined closed linear subspace of $L^2(G,\tau)$ on which G acts continuously and isometrically by the usual left regular action L. Thus, G acts on $L^2(KerD)$ by a unitary representation. In the key work [2] it is shown that the representation of G in $L^2(KerD)$ is a finite sum of irreducible square integrable representations of G. Since $\bar{D}$ is an elliptic (with real analytic coefficients) linear differential operator we have that $L^2(KerD)$ is contained in the space of real analytic sections of the bundle $E \to G/K$. Let $(\tau_\star, W)$ denote the restriction of τ to the subgroup $H \cap K$. Let

$$F := H \times_{\tau_\star} W \longrightarrow H/(H \cap K)$$

denote the associated $H-$homogeneous, Hermitian bundle over $H/(H \cap K)$. Owing to our choice, $H/(H \cap K)$ can be thought as the orbit of H through the point eK on G/K and F as a subbundle of E over this orbit, and hence we may restrict smooth sections of E over G/K to sections of F over $H/(H \cap K)$. Let

$$r : C^\infty(G, \tau) \longrightarrow C^\infty(H, \tau_\star)$$

denote the restriction map. The first result of this paper is:

Theorem 1.1. — *Let G, D, H be as before, then*
1) $r(L^2(KerD)) \subset L^2(H, \tau_\star)$.
2) $r : L^2(KerD) \longrightarrow L^2(H, \tau_\star)$ is a continuous map.
3) The generalized Berezin transform, $rr^\star$, is a continuous linear operator on $L^2(H, \tau_\star)$.

We also show that,

Theorem 1.2. — *Besides, whenever $L^2(KerD)$ is a sum of integrable representation of G, then the generalized Berezin transform, $rr^\star$, is a continuous linear operator on $L^p(H, \tau_\star)$, $1 \leqslant p \leqslant \infty$.*

Our hypothesis on H implies that H is invariant under the Cartan involution associated to K. Thus, we have the $Ad(H \cap K)-$invariant decompositions $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{s}$, $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{q}$, and $\mathfrak{q} = \mathfrak{q} \cap \mathfrak{k} \oplus \mathfrak{q} \cap \mathfrak{s}$. For each nonnegative integer m let $S^m(\mathfrak{q} \cap \mathfrak{s})$ denote the $m^{th}-$symmetric power of $\mathfrak{q} \cap \mathfrak{s}$. Thus, $S^m(\mathfrak{q} \cap \mathfrak{s}) \otimes W$ is an $H \cap K-$module. A basic idea in branching theory is to consider normal derivatives corresponding to the immersion $H/H \cap K \to G/K$. Using this we may show the following,

Theorem 1.3. — *There exists an injective, linear $H-$map, $f \to (f_m)_{m \geqslant 0}$ from*

$$L^2(KerD) \text{ into } \oplus_{m \geqslant 0} L^2(H \times_{H \cap K} (S^m(\mathfrak{s} \cap \mathfrak{q}) \otimes W))$$

so that for each $m \geqslant 0$, the linear map

$$L^2(KerD) \ni f \to f_m \in L^2(H \times_{H \cap K} (S^m(\mathfrak{s} \cap \mathfrak{q}) \otimes W))$$

is continuous.

2. Proof of Theorem 1 and Theorem 2

Recall the restriction r defined in the Introduction; this is the main object of study, as well as the associated Berezin transform $rr^\star$. In order to verify that $r^\star$ is defined, we show the following:

i : If $f \in L^2(KerD)$ is a $K-$finite function, then $r(f) \in L^2(H, \tau_\star)$

ii : Let $\mathcal{D} := \{f \in L^2(KerD) : r(f) \in L^2(H, \tau_\star)\}$, then $r : \mathcal{D} \longrightarrow L^2(H, \tau_\star)$ is a closed densely defined linear transformation.

To justify the above statements we recall the reproducing kernel for $L^2(KerD)$. Since D is an elliptic operator, L^2 convergence in $L^2(KerD)$ implies uniform convergence in the induced topology by $C^\infty(G, \tau)$ ([10], Theorem 52.1) Thus, point evaluations are continuous linear functionals in $L^2(KerD)$. Therefore, the orthogonal projection of $L^2(G, \tau)$ onto $L^2(KerD)$ is an integral operator given by a smooth kernel

$$k : G \times G \longrightarrow End_{\mathbb{C}}(W).$$

We have,

$$(f(z), v)_W = (f, k(z, \cdot)v)_{L^2(G,\tau)}, \; f \in L^2(KerD), \; z \in G, \; v \in W. \tag{1}$$

Here, $(.,.)_Z$ denotes the inner product in the Hilbert space Z.

$$k(x, y)^\star = k(y, x), x, y \in G. \tag{2}$$

For a proof of these facts, we refer to [8]. Since the orthogonal projection commutes with the action of G, there exists a smooth function $k : G \to End_{\mathbb{C}} W$ so that

$$k(x, y) = k(x^{-1}y), x, y \in G. \tag{3}$$

Actually, k is the vector in $L^2(KerD)$ that represents the linear functional point evaluation at the identity of e of G. Since point evaluation at the identity is a $K-$finite linear functional and $L^2(KerD)$ is an admissible representation (see [5])there exists nice estimates for k as well for any $K-$finite element $f \in L^2(KerD)$ which we now describe.

For a semisimple Lie group G, let

$$\Xi_G(x) = \int_K e^{-\rho_G(H(xk))} dk$$

denote the Harish-Chandra $\Xi-$function, see [5] page 186 (with a different notation). Relative to an Iwasawa decomposition (that we fix) $G = KAN$, we have the projection H given by $x \in Ke^{H(x)}N, x \in G$. We recall that

$\Xi_G \in L^{2+\gamma}(G)$ for every $\gamma > 0$. Since $L^2(KerD)$ is a finite sum of square integrable representations, and both k, f are $K-$finite elements of $L^2(KerD)$, it follows from [11] and the equality

$$\|f(x)\|^2 = \sum_i (f, L_x(k(\cdot)(w_i)))_{L^2(G,\tau)}$$

for an orthonormal basis w_i of W, that we have an estimate of the pointwise norm of $f(x)$, coming from the asymptotics of matrix coefficients of the representation, namely: for any $q \geqslant 0$, and for $\epsilon > 0$ sufficiently small, there exists $0 \leqslant C_f < \infty$ so that

$$\|f(x)\| \leqslant C_f \, \Xi_G^{1+\epsilon}(x)(1 + \|x\|)^q, \ x \in G. \tag{2.a}$$

(In fact we only need this equation for $q = 0$; such a polynomial term is relevant when seeking the best possible exponent.) The norm $\|\cdot\|$ is a K-biinvariant norm on the group, coming from a Euclidean norm on the Cartan complement to the Lie algebra of K, defined as in [5] page 256, $\|x\| =: \|Y\|$, if $x = k_1 exp(Y), k_1 \in K, Y \in \mathfrak{s}$. Besides, whenever $L^2(KerD)$ is a sum of integrable representations, as explained in [11] or in [5] page 256, we find the analogous estimate, with $q' \geqslant 0$,

$$\|f(x)\| \leqslant C'_f \, \Xi_G^{2+\epsilon}(x)(1 + \|x\|)^{q'}, \ x \in G. \tag{2.b}$$

We now show that these estimates lead to the following:

Lemma 2.1. — *Assume, $f \in L^2(KerD)$ is a $K-$finite element, then there exists $0 < \delta < 1$ so that*

$$f \in L^p(H, \tau_\star) \ for \ 2 - \delta \leqslant p \leqslant 2 \tag{p.1}$$

Besides, if $L^2(kerD)$ is a sum of integrable representations of G, then

$$f \in L^2(H, \tau_\star) \cap L^1(H, \tau_\star) \tag{p.2}$$

Proof: (Here we follow [9].) We will show that the restriction of f to the center of H is an integrable function on the center of H, and when H is semisimple, that $f \in L^{2-\delta}(H, \tau_\star)$ for δ small and positive. We fix compatible Iwasawa decompositions $G = KAN, H = (H \cap K)A_H N_1$, $A_H \subset A$, $N_1 \subset N$ (in particular we have chosen appropriately compatible orderings of the restricted roots) and use the integral formula associated to the Cartan decomposition $H = (H \cap K)A_H(H \cap K)$. In particular, we denote the corresponding density function on A_H by Δ as usual, see e.g. [5]. Let $\mathfrak{a}_H^+$ be the closed Weyl chamber in $\mathfrak{a}_H$ associated to N_1. Let $C_1, \cdots, C_S$ be the open $G-$Weyl chambers in $\mathfrak{a}$ so that $\mathfrak{a}_H^+ \cap \bar{C}_j$ has non-empty interior in $\mathfrak{a}_H^+$. Hence, $\mathfrak{a}_H^+ = \cup_j(\mathfrak{a}_H^+ \cap \bar{C}_j)$. Let $\rho_H(Y) = \frac{1}{2}tr(ad(Y)|_{\mathfrak{n}_1})$ and denote by

$$\rho_j(Y) = \frac{1}{2}tr(ad(Y)|_{\sum_{\alpha:\alpha(C_j)>0}\mathfrak{n}_\alpha})$$

the $\rho-$element in $\mathfrak{a}^\star$ corresponding to C_j. Here, positive restricted root spaces in $\mathfrak{g}$ are denoted $\mathfrak{n}_\alpha$, and we also denote similarly $\mathfrak{n}_{1\,\beta}$ root spaces in $\mathfrak{h}$, the Lie algebra of H. The slight abuse of notation $\alpha(C_j) > 0$ means that α takes a positive value on C_j. Hence, for each j, we have the inequality

$$\rho_H(Y) - \rho_j(Y) \leqslant 0, \forall\, Y \in \mathfrak{a}_H^+ \cap \bar{C}_j. \tag{2.c}$$

Indeed, if $\alpha \in \Phi(\mathfrak{g},\mathfrak{a})$ and $\alpha(C_j) > 0$, then the restriction of α to $\mathfrak{a}_H$ is either zero, or a restricted root for $(\mathfrak{n}_1, \mathfrak{a}_H)$, or a nonzero linear functional on $\mathfrak{a}_H$, and in general we cannot say that the restriction of α to $\mathfrak{a}_H$ takes on nonnegative values on $\mathfrak{a}_H^+$. But, more importantly, we can on the other hand say , that if $\beta \in \Phi(\mathfrak{n}_1, \mathfrak{a}_H)$ then β is the restriction of restricted roots $\alpha_1, \cdots, \alpha_R$. Hence, for each $1 \leqslant s \leqslant R$, we have that $\alpha_s(\bar{C}_j \cap \mathfrak{a}_H^+) > 0$. The choice of C_j implies that $\alpha_s(C_j) > 0$ and that

$$\mathfrak{n}_{1\,\beta} \subset \oplus_{\{\alpha\in\Phi(\mathfrak{g},\mathfrak{a}):\alpha(C_j)>0,\, \alpha|_{\mathfrak{a}_H}=\beta\}}\ \mathfrak{n}_\alpha. \tag{2.d}$$

Note that ρ_H is defined as a sum of positive traces over the spaces on the left side in equation $(2.d)$, whereas the ρ_j consists of sums over spaces on the right side of $(2.d)$, and that the definition of C_j ensures that the value of ρ_j is larger than the value of ρ_H on $\mathfrak{a}_H^+ \cap \bar{C}_j$. Thus, the inequality $(2.c)$ follows - it is one of the main points in our argument. Now $(2.a)$ and the above inequality, justify the steps in

$$\begin{aligned}
\int_H \|f(h)\|^{2-\delta}dh =& \int_{A_H^+} |\Delta(a)|^2 \int_{H\cap K}\int_{H\cap K} \|f(k_1ak_2)\|^{2-\delta}dk_1dk_2da \\
&<<\sum_j \int_{\bar{C}_j\cap\mathfrak{a}_H^+} e^{2\rho_H(Y)}e^{-(1+\epsilon)(2-\delta)\rho_j(Y)}(1+\|Y\|)^{q(2-\delta)}dY \\
&\leqslant\sum_j \int_{\bar{C}_j\cap\mathfrak{a}_H^+} e^{2\rho_H(Y)}e^{-(1+\epsilon)(2-\delta)\rho_H(Y)}(1+\|Y\|)^{q(2-\delta)}dY \\
&=\sum_j \int_{\bar{C}_j\cap\mathfrak{a}_H^+} e^{2\rho_H(Y)-2\rho_H(Y)}e^{(-2\epsilon+\delta(1+\epsilon))\rho_H(Y)}(1+\|Y\|)^{q(2-\delta)}dY \\
&< \infty
\end{aligned}$$

for δ positive and small enough. This shows that f is in $L^{2-\delta}(H)$ for δ small enough. The same formalism shows that whenever $L^2(KerD)$ is a sum of integrable representations, then f is integrable on H, as claimed in (p.2). When the center of H has a split factor, on the intersection of the split factor with the closure of a Weyl chamber C in $\mathfrak{a}$ we have that

$\|f(exp(Y))\|$ is bounded by $e^{-\rho_C(Y)}(1+\|Y\|)^q$. (Here ρ_C is the rho-element corresponding to C; actually we could have just considered $q = 0$, the polynomial term being necessary when seeking the best possible exponent.) Thus f is integrable when restricted to the center of H.

□

The above consideration and Lemma 1 show that for any $f \in L^2(KerD)$ which is $K-$finite, we have that $r(f) \in L^2(H, \tau_\star) \cap L^\infty(H, \tau_\star)$. A proof that $(r, \mathcal{D})$ is a closed linear transformation follows from that L^2-convergence in $L^2(KerD)$ implies uniform convergence on compact sets. This concludes the proof of **i**, **ii**.

Therefore, $(r, \mathcal{D})$ is a closed densely defined linear transform, let $r^\star$ its adjoint. We want to compute the linear operator $rr^\star$ evaluated at g in the domain of $r^\star$. For this we note that for each $z \in H$, $k(z, \cdot)v$ belongs to the domain of r, and we recall the Kunze-Stein phenomenon, see [3]: for H a connected semisimple Lie group with finite center, we have that

$$L^p(H) * L^2(H) \subset L^2(H)$$

whenever $1 \leqslant p < 2$. Hence, Lemma 1 and the Kunze-Stein phenomenon imply that the map

$$f \to \int_H k(h, z)f(h)dh$$

defines a bounded linear operator on $L^2(H, \tau_\star)$. We now verify that

$$(rr^\star)(g)(z) = r^\star(g)(z) = \int_H k(h, z)g(h)dh,\ z \in H, \tag{4}$$

for g in the domain of $r^\star$. In fact, the equality (1) applied to $r^\star(g)$ yields

$$(r^\star(g)(z), v)_W = (r^\star(g), k(z, \cdot)v)_{L^2(G,\tau)}$$

Since we have

$$\begin{aligned}
(r^\star(g), k(z, \cdot)v)_{L^2(G,\tau)} &= (g, r(k(z, \cdot)v)_{L^2(H,\tau_\star)} \\
&= \int_H (g(h), k(z, h)v)_W dh \\
&= \int_H (k(z, h)^\star g(h), v)_W dh \\
&= \int_H (k(h, z)g(h), v)_W dh \\
(r^\star(g)(z), v)_W &= (\int_H k(h, z)g(h)dh, v)_W.
\end{aligned}$$

We now conclude the proof of Theorem 1. Indeed, Lemma 1, (4) and the Kunze-Stein phenomena imply that $rr^\star$ extends to a bounded linear operator on $L^2(H,\tau_\star)$. Since, r is closed, the polar decomposition of r is valid, that is, for U a partially unitary linear operator (partial isometry), we have

$$r = (rr^\star)^{1/2}U.$$

Thus, r extends to a bounded linear transformation $\tilde{r}$ from $L^2(KerD)$ into $L^2(H,\tau_\star)$. Once again, the fact that L^2- convergence in $L^2(KerD)$ implies uniform convergence on compact sets, forces that $\tilde{r}$ is equal to r. This concludes the proof of Theorem 1. The proof of Theorem 2 follows the same line of thought and we use (p.2) instead of (p.1).

3. Proof of Theorem 3

Let $\mathcal{S}(\mathfrak{g})$ (resp. $\mathcal{U}(\mathfrak{g})$)be symmetric algebra of $\mathfrak{g}$ (resp. the universal enveloping algebra of $\mathfrak{g}$). Let $\lambda : \mathcal{S}(\mathfrak{g}) \to \mathcal{U}(\mathfrak{g})$ be the symmetrization. For any $Y \in \mathcal{U}(\mathfrak{g}), f \in C^\infty(G), g \in G$,

$$R_D Y(f)(g) = \lim_{t\to 0} \frac{f(gexp(tY)) - f(g)}{t}$$

defines infinitesimal right translation by Y. Let

$$r_m : C^\infty(G,\tau) \to C^\infty(H, Hom_{\mathbb{C}}(S^m(\mathfrak{s}\cap\mathfrak{q}), W))$$

be the linear map defined by the rule

$$r_m(f)(h)(X_1,\cdots,X_m) = (R_{\lambda(X_1,\cdots,X_m)}f)(h).$$

The action, via the Adjoint representation of $H\cap K$ in $\mathfrak{s}\cap\mathfrak{q}$, gives rise to a representation of $H\cap K$ in $Hom_{\mathbb{C}}(S^m(\mathfrak{s}\cap\mathfrak{q}), W)$. We denote this representation by τ_m. It readily follows that

$$r_m(f)(hk) = \tau_m(k^{-1})r_m(f)(h), h\in H, k \in H\cap K.$$

Thus, r_m maps $C^\infty(G,\tau)$ into $C^\infty(H,\tau_m)$. For any pair G, H and $L^2(KerD)$ we may prove, by arguments similar to those already used, the following three points below;

(1) r_m is a closed densely defined linear transformation from $L^2(KerD)$ into $L^2(H,\tau_m)$ whose domain contains the $K-$finite vectors. Indeed, this also follows from the estimate in Corollary 7.4 in [11] and the proof of Lemma 1. Here we obtain it from the formula in (2) below, and the corresponding boundedness of the restriction r_m derived in a similar way from the estimates of k_m.

(2) r_m extends to a continuous linear map from $L^2(KerD)$ into $L^2(H, \tau_m)$. In fact, as in the proof of Theorem 1, we have that $r_m r_m^\star$ is an integral operator given by a kernel function analogous to the case of no derivatives. In the case of derivatives, the kernel is

$$k_m(x,y)(Y \otimes v) = k(x,y)(\dot{L}_{\lambda(Y)}(v)).$$

Here, $\dot{L}$ denotes the differential of the representation L.

(3) The map

$$f \to (r_m(f))_{m \geqslant 0} \text{ from } L^2(KerD) \text{ into } \oplus_{m \geqslant 0} L^2(H, S^m(\mathfrak{q} \cap \mathfrak{s}) \otimes W))$$

is injective and each component is bounded. Hence Theorem 3 is proved.

As a final remark we note, that with Theorem 3 we now have in principle a way of analyzing the restriction to H of the original unitary representation; one open problem remains to find the image of the restriction map of the normal derivatives, and another to decompose explicitly the H - homogeneous vector bundles (which in some sense is covered by Harish-Chandra's Plancherel Theorem.)

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