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# A BOUNDEDNESS THEOREM FOR MORPHISMS BETWEEN THREEFOLDS

by E. AMERIK, M. ROVINSKY and A. VAN DE VEN

## 0. Introduction.

Let $X$ and $Y$ be smooth projective $n$-folds (over a field of characteristic zero) and $f : X \to Y$ a surjective morphism. One can ask the following:

General question. — *Which varieties $Y$ have the following property:*

($*$) *For any $X$, the degree of $f : X \to Y$ is bounded in terms of discrete invariants of $X$ and $Y$?*

If we take $Y$ of general type, then by the result of [KO], for any $X$ there is only a finite number of maps $f : X \to Y$. Some bounds for $\deg(f)$ in terms of the discrete invariants of $X$ and $Y$ are obtained in works of T. Bandman, G. Dethloff and D. Markushevich ([BD], [BM]).

If $Y$ is not of general type, there are very few results in this direction and the question seems to be rather difficult. Obviously, there are varieties which do not have the property ($*$) (abelian varieties, projective spaces), but one may hope that they can be classified.

The following conjecture seems to be a good starting point:

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Conjecture 0.1 ([A], p. 196). — *Let $X$ and $Y$ be smooth $n$-dimensional complex projective varieties such that $b_2(X) = b_2(Y) = 1$. If $Y \neq \mathbb{P}^n$ (or if $Y$ is of genus at least two in the 1-dimensional case), then the degree of a map $f : X \to Y$ is bounded in terms of the discrete invariants of $X$ and $Y$.*

As a motivation for this conjecture, we mention that if $Y$ is a smooth quadric of dimension $n \geq 3$, then for any smooth projective $n$-fold $X$ such that the Néron-Severi group of $X$ is of rank one, the degree of a map $f : X \to Y$ is bounded by certain invariants of $X$ ([S] for $n = 3$, [A1] for arbitrary $n$).

For dimensions 1 and 2 the conjecture is very easy (a surface with $b_2 = 1$ is either the projective plane or of general type). The first author ([A]) has obtained a proof in the case where $Y$ is a Fano threefold with a very ample generator of the Picard group.

The main purpose of this note is to complete the proof of the conjecture in dimension three, that is, to deal with maps onto other Fano threefolds and onto varieties with trivial (in $H^2(Y, \mathbb{R})$) canonical class:

Theorem 0.2. — *Conjecture 0.1 is true for $n = 3$.*

The paper is organized as follows: in the first paragraph, a Hurwitz-type inequality (Corollary 1.2) is proved. Then in the second paragraph this result is applied to maps onto varieties with trivial canonical class and the relevant Fano threefolds. In this way the problem reduces to a problem about torus quotients which is handled in the last section.

## 1. A Hurwitz-type formula.

Lemma 1.1. — *Let $f : X \to Y$ be a finite morphism between smooth projective varieties. Let $L$ be a line bundle on $Y$ such that $\Omega_Y(L)$ is globally generated. Then for a generic section $s \in H^0(Y, \Omega_Y(L))$ the induced section $t_s \in H^0(X, \Omega_X(f^*L))$ has only isolated zeroes.*

*Proof.* — Suppose first that $\dim X = \dim Y = 2$. With respect to the local coordinates on $X$ and $Y$ the map $\phi : s \to t_s$ is locally given by the Jacobian matrix. So if for a generic $s$, $t_s$ vanishes on a curve, this curve is a component $R^0$ of the ramification divisor (and all the $t_s$ vanish on $R^0$). We will denote $f^*s$ by the same letter $s$.

The restriction $s_{R^0}$ of $s$ to $R^0$ must therefore lie in the subsheaf of rank one

$$\mathrm{Ker}(f^*(\Omega_Y(L))|_{R^0} \to \Omega_X(f^*L)|_{R^0}),$$

which is a contradiction because $f^*(\Omega_Y(L))|_{R^0}$ is generated by the restrictions $s_{R^0}$.

The proof in the general case goes along the same lines. Let $X$, $Y$ be $m$-dimensional. Denote by $R_i$ the locus where $rk(f) \leq i$. As $f$ is a finite morphism,

$$\dim R_i \leq i.$$

Now suppose that for a general $s \in H^0(Y, \Omega_Y(L))$, the induced section $t_s$ vanishes along a curve $C_s$. Let $j$ be such that $C_s \subset R_j$ and $C_s \not\subset R_{j-1}$ for a generic $s$. We may suppose that $R_j$ is irreducible (otherwise we take an irreducible component containing $C_s$ for a general $s$).

Now $s$ induces a section $s_{R_j}$ of the sheaf

$$A_j = f^*(\Omega_Y(L))|_{R_j}/B_j$$

where

$$B_j = \mathrm{Ker}(f^*(\Omega_Y(L))|_{R_j} \to \Omega_X(f^*L)|_{R_j}).$$

Obviously the section $s_{R_j}$ vanishes on $C_s$.

The sheaf $A_j$ on $R_j$ is of rank $j$, it is locally free outside $R_{j-1}$ and generated by the sections $s_{R_j}$ for all $s \in H^0(Y, \Omega_Y(L))$. Hence by a Bertini-type theorem of Kleiman ([K]), outside of $R_{j-1}$ the generic $s_{R_j}$ should vanish at most at points, *i.e.* $C_s \subset R_{j-1}$, a contradiction.

COROLLARY 1.2 (the "Hurwitz formula"). — *In the situation of Lemma 1.1,*

$$\deg(f)c_{\mathrm{top}}(\Omega_Y(L)) \leq c_{\mathrm{top}}(\Omega_X(f^*L)).$$

*Proof.* — It is always possible to choose an $s$ with isolated zeroes only, all outside the branch locus of $f$.

*Remark.* — In the same way, one can relate the intermediate Chern classes of $X$ and $Y$ by showing that for general sections $s_1, \ldots, s_k$ of $\Omega_Y(L)$, the linear dependency locus of the induced sections of $\Omega_X(f^*L)$ has correct codimension. The analogue of the lemma in this situation is as follows: $c_i(\Omega_X(f^*L)) - f^*c_i(\Omega_Y(L))$ can be represented by an effective cycle in the Chow group $A^i(X)$.

## 2. Applications to the boundedness questions.

There are several applications of this lemma. The first one is to morphisms onto varieties with trivial first Chern class:

PROPOSITION 2.1. — *Let $X$ be a smooth projective variety of dimension $n$ and such that $NS(X) = \mathbb{Z}$, let $Y$ be a smooth projective variety of dimension $n$ such that $c_1(Y) = 0$ in $H^2(Y, \mathbb{R})$. If $f : X \to Y$ is a finite morphism, then the degree of $f$ is bounded in terms of the discrete invariants of $X$ and $Y$ unless $Y$ is a quotient of a torus by a freely acting finite group.*

*Proof.* — Obviously we have $NS(Y) = \mathbb{Z}$. Denote by $H_Y$ and $H_X$ ample generators of $NS(Y)$, $NS(X)$. Then there is a number $m \in \mathbb{Z}$ such that $f^*H_Y = mH_X$, and we want to know when this $m$ is bounded. For $L$ such that $\Omega_Y(L)$ is globally generated, let $\ell$ be such that $L \equiv \ell H_Y$ in the Néron-Severi group. Applying Corollary 1.2, one gets

$$m^n \frac{H_X^n}{H_Y^n}(\ell^n H_Y^n + \ell^{n-1}H_Y^{n-1}c_1(\Omega_Y^1) + \ell^{n-2}H_Y^{n-2}c_2(\Omega_Y^1) + \cdots + c_n(\Omega_Y^1))$$
$$\leq (\ell m)^n H_X^n + (\ell m)^{n-1}H_X^{n-1}c_1(\Omega_X^1) + (\ell m)^{n-2}H_X^{n-2}c_2(\Omega_X^1) + \cdots + c_n(\Omega_X^1).$$

If $m$ is not bounded, then for *any* such $L$

$$\ell^{n-2}H_Y^{n-2}c_2(\Omega_Y^1) + \cdots + c_n(\Omega_Y^1) \leq 0.$$

Replacing $L$ by $kL$ for $k \gg 0$ if necessary, we find that either $m$ is bounded, or $c_2(\Omega_Y^1)H_Y^{n-2} \leq 0$.

According to [Kob], Chapter 4, section 4, if $Y$ is Kähler-Einstein with trivial first Chern class and if $\Phi$ is the Kähler-Einstein form, then always

$$\int_Y c_2(Y) \wedge \Phi^{n-2} \geq 0$$

and

$$\int_Y c_2(Y) \wedge \Phi^{n-2} = 0$$

implies that $Y$ is flat. But thanks to Yau's theorem ([Y]) we know there is a Kähler-Einstein metric on $Y$ such that the class of $\Phi$ is proportional to that of $H_Y$. Therefore $c_2(\Omega_Y^1)H_Y^{n-2} = 0$, our variety $Y$ is flat and hence by the classical Bieberbach's theory is a quotient of a torus.

Another application concerns maps onto Fano threefolds. In the paper [A], it was shown that if $X$ is a threefold with $b_2(X) = 1$ and $Y$ is a Fano threefold with $b_2(Y) = 1$ such that the ample generator $H_Y$ of $\mathrm{Pic}(Y)$ is very ample, then the degree of a map $f : X \to Y$ is bounded in terms of the discrete invariants of $X$ and $Y$. We will use Corollary 1.2 to prove boundedness for morphisms onto other Fano threefolds.

There are four types of Fano threefolds $Y$ with not very ample generator of $\mathrm{Pic}(Y)$ ([I]):

of index 2:

1) A double cover of $\mathbb{P}^3$ ramified along a quartic; this is a hypersurface of degree 4 in the weighted projective space $\mathbb{P}(1,1,1,1,2)$.

2) A double cover of a cone over the Veronese surface ramified along a cubic section of this cone; this is a hypersurface of degree 6 in the weighted projective space $\mathbb{P}(1,1,1,2,3)$.

of index 1:

3) A double cover of $\mathbb{P}^3$ ramified along a sextic; this is a hypersurface of degree 6 in the weighted projective space $\mathbb{P}(1,1,1,1,3)$.

4) A double cover of a quadric ramified along a quartic section of this quadric.

PROPOSITION 2.2. — *If $X$ is a threefold with $b_2(Y) = 1$ and $Y$ is one of the four threefolds mentioned above, then the degree of a map $f : X \to Y$ is bounded in terms of the discrete invariants of $X$ and $Y$.*

*Proof.* — Note that since it is shown that the degree of a map from a threefold with $b_2 = 1$ to a 3-quadric is bounded ([A1], [S]), we need not to consider the case 4.

It follows from Corollary 1.2 (as in the proof of Proposition 2.1) that it is sufficient to show that there is an integer $\ell$ with $\Omega_Y(\ell H_Y)$ globally generated and

$$c_3(\Omega_Y) + \ell H_Y c_2(\Omega_Y) + \ell^2 H_Y^2 c_1(\Omega_Y) > 0,$$

so in fact one is interested in finding the smallest possible $\ell$ such that $\Omega_Y(\ell H_Y)$ is globally generated.

Note that all the weighted projective spaces which appear in 1), 2), 3), have isolated singularities, and our Fano hypersurfaces cannot pass through these (otherwise they would be singular).

On the smooth part $U$ of a weighted projective space $\mathbb{P}(e_0, e_1, \ldots, e_4)$ we have an exact sequence

$$0 \to \mathcal{O}_U \to \oplus \mathcal{O}_U(e_i) \to T_U \to 0$$

where $T_U$ is the tangent bundle. Using the fact that $\Omega_Y$ is a quotient of

$$\Omega_U|Y = \Lambda^3 T_U|Y \otimes \det \Omega_U|Y,$$

we find that:

— in the first case $\Omega_Y(3H_Y)$ is globally generated,

— in the second case $\Omega_Y(7H_Y)$ is globally generated,

— in the third case $\Omega_Y(4H_Y)$ is globally generated.

So for example in the second case, where $K_Y = -2H_Y$, $H_Y^3 = 1$ and $b_3(Y) = 42$ (see [D], section 4.3, for the Hodge numbers of weighted hypersurfaces), we find

$$c_3(\Omega_Y) + \ell H_Y c_2(\Omega_Y) + \ell^2 H_Y^2 c_1(\Omega_Y) = 38 + 84 - 98 > 0$$

(note that $H_Y c_2(\Omega_Y) = \frac{1}{2} c_1(Y) c_2(Y) = 12\chi(\mathcal{O}_Y) = 12$), and therefore $\deg(f)$ is bounded. In two other cases the same computation also yields the boundedness.

This completes the above-mentioned result of [A] on boundedness of degree of maps onto Fano threefolds with $b_2 = 1$. Notice that the argument presented here does not "cancel" the (much more lengthy) proof in [A], as there are also families of Fano threefolds $Y$ such that

$$c_3(\Omega_Y) + \ell H_Y c_2(\Omega_Y) + \ell^2 H_Y^2 c_1(\Omega_Y) \leq 0$$

for all $\ell$ such that $\Omega_Y(\ell H_Y)$ is globally generated, or even for all $\ell$ for which $\Omega_Y(\ell H_Y)$ has sections. It would be interesting to know whether there is some "geometric" difference between Fano varieties for which the above expression is positive or non-positive.

## 3. On torus quotients.

Proposition 2.1 solves completely the question of boundedness for the degree of a map $f : X \to Y$ where $X$, $Y$ have Néron-Severi group of rank one and $Y$ has numerically trivial canonical class. Namely, $\deg(f)$ is bounded if $Y$ is not a flat manifold (*i.e.* torus quotient by a finite group acting freely) and not bounded otherwise. To deal with Conjecture 0.1, we must understand what happens if one restricts to varieties with $b_2 = 1$, in other words:

*Does there exist an algebraic torus quotient $Y = G\backslash A$ by a freely acting finite group $G$, such that $b_2(Y) = 1$?*

If $\dim(A)=2$, this is of course impossible because $b_2(Y) = 1$ implies $b_1(Y) = 0$, whereas we must have $\chi(\mathcal{O}_Y) = 0$. In this paragraph we prove that this is also impossible in dimension 3, thus finishing the proof of Theorem 0.2.

PROPOSITION 3.1. — *A quotient $Y$ of a 3-dimensional abelian variety $A$ by a finite, freely acting group $G$, cannot have $b_2 = 1$.*

*Proof.* — Suppose $b_2(Y) = 1$. Then

1. *The representation $\rho$ of $G$ in the tangent space $V = T_0A$ is irreducible:* it is easy to see that for any irreducible representation $W$ of $G$, $W \otimes \overline{W}$ has a one-dimensional fixed subspace, so if $V$ is not an irreducible representation, $b_2(Y) \neq 1$.

2. Then it follows that *$A$ is isogeneous to the cube of an elliptic curve $E$*. Indeed, if $A$ is isogeneous to a product of non-isogeneous simple abelian varieties, then our representation is obviously not irreducible; if $A$ is simple 3-dimensional in characteristic zero, then $\mathrm{Hom}(A, A)$ is commutative (see for example [M], p. 182).

*In 3, 4, 5, 6 we prove that $G$ contains $A_4$ as a normal subgroup (in fact $G$ is $A_4$ or $S_4$):*

3. For any element $g \in G$ we have $\det(\rho(g)) = 1$ since $h^{3,0}(Y) = 1$ from $\chi(\mathcal{O}_Y) = 1$. As all elements are of finite order, all eigenvalues of $\rho(g)$ are roots of unity. Notice that one of them must be 1: otherwise $\rho(g) - I$ is invertible, that is, for any $h \in A$ the transformation $y \to \rho(g)y + h$ of $A$ has a fixed point, and so the action of $G$ is never free. Therefore $\mathrm{Tr}(\rho(g)) \in \mathbb{R}$. As $\mathrm{Tr}(\rho(g))$ also is an endomorphism of $E$, we must have $\mathrm{Tr}(\rho(g)) \in \mathbb{Z}$, which is the same as to say that $g$ as an endomorphism of $V$ has order 1, 2, 3, 4 or 6.

4. We may assume that the representation of $G$ on $V$ is exact, as otherwise we can factorize $A$ by the translation subgroup of $G$. So we will

write simply $g$ instead of $\rho(g)$ when there is no danger of confusion. Let $N_i$ be the number of elements of order $i$ in $G$. As $V$ is irreducible, we have

$$\sum_{g\in G} \mathrm{Tr}(g)\overline{\mathrm{Tr}(g)} = \mathrm{ord}(G)$$

or equivalently

$$9 + N_2 + N_4 + 4N_6 = 1 + N_2 + N_3 + N_4 + N_6,$$

which means that $N_3 = 3N_6 + 8 \neq 0$.

Also, as there are no fixed vectors in $V$, we have

$$\sum_{g\in G} g = 0,$$

so

$$\sum_{g\in G} \mathrm{Tr}(g) = 3 - N_2 + N_4 + 2N_6 = 0$$

and therefore $N_2 \neq 0$.

5. Consider a Sylow 3-subgroup of $G$: as $N_2 \neq 0$, its action cannot be irreducible (otherwise $9 = \mathrm{ord}(G)$ but $N_3 \geq 8$). On the other hand, this subgroup does not have irreducible 2-dimensional representations (the dimension of an irreducible representation must divide the order of the group). This means that the action of this subgroup can be diagonalised and one verifies (taking into account that each $g$ has $\det(g) = 1$ and $g - I$ is never invertible) that it is $\mathbb{Z}/3\mathbb{Z}$.

6. Now there are no elements of order 6 in $G$: otherwise one verifies $N_6 \geq N_3$ (all the elements of order three are obtained as squares of elements of order six, because all Sylow 3-subgroups are conjugated) which is impossible. Hence $N_3 = 8$. As no element of order 2 or 4 can therefore commute with an element of order 3, we get that the order of a Sylow 2-subgroup is at most 8. Similar elementary considerations show that there are only two possibilities for $G$:

a) $\mathrm{ord}(G) = 12$, the Sylow 2-subgroup is normal and isomorphic to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$: $G$ is the (noncommutative) semidirect product of the Sylow 2-subgroup and any Sylow 3-subgroup, *i.e.* this is the group $A_4$ of even permutations on 4 elements.

b) $\mathrm{ord}(G) = 24$, so $N_2 = 9$ and $N_4 = 6$; then each Sylow 2-subgroup is isomorphic to the (noncommutative) semidirect product of $\mathbb{Z}/4\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z}$, there are exactly three Sylow 2-subgroups $P_1, P_2, P_3$ and there is a normal subgroup $C$ isomorphic to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ which is the intersection of these Sylow subgroups. There is also a subgroup isomorphic to $S_3$ which is generated by an element $a$ of order three and an element $x \in P_1$ such that $xax^{-1} = a^2$: one checks that $x \notin C$ and the group $G$ which is a semidirect product of $C$ and $S_3$ is isomorphic to $S_4$.

In both cases, there must be a free action of $A_4$ on $A$ which is irreducible on the tangent space.

7. Now we have to show that the action of $A_4$ on $V$ cannot be lifted to a free action of $A_4$ on $A$. Indeed, this lifting is a function $f : A_4 \to A$ which satisfies

$$f(ab) = af(b) + f(a),$$

*i.e.* is a cocycle in $C^1(A_4, A)$ (where the action of $A_4$ on $A$ is of course not our free action but the "linear" action induced by the action on $V$). Note that the coboundaries are of the form $b(a) = ah - h$ for some $h \in A$; the transformation $x \to ax + ah - h$ always has a fixed point. Therefore it is enough to prove the following:

CLAIM. — *For any cocycle $\eta \in H^1(A_4, A)$, there exists a subgroup $H$ of $A_4$ such that the restriction of $\eta$ to $H^1(H, A)$ is zero.*

This is done as follows:

8. Note that for our linear action of $C = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \subset A_4$ on $A$, *the invariants $H^0(C, A)$ are elements of 4-torsion.* More generally, for any irreducible exact representation $V$ of a finite group $G$, a $G$-invariant lattice $\Lambda$ and a normal subgroup $H \subset G$, the invariants $H^0(H, V/\Lambda)$ are $\mathrm{ord}(H)$-torsion.

Indeed, $H$ does not fix any vector of $V$ (as it is a normal subgroup of $G$ and $V$ is $G$-irreducible). From the short exact sequence of $G$-modules

$$0 \to \Lambda \to V \to V/\Lambda \to 0$$

we get

$$H^0(H, V/\Lambda) \subset H^1(H, \Lambda).$$

Since $H^1(H, \Lambda)$ is annihilated by the multiplication by $\mathrm{ord}(H)$, so is $H^0(H, V/\Lambda)$.

9. The *Serre-Hochschild spectral sequence* for $C, A_4$ and the quotient $B = A_4/C$:

$$E_2^{pq} = H^p(B, H^q(C, A)) => H^{p+q}(A_4, A)$$

gives

$$0 \to H^1(B, H^0(C, A)) \to H^1(A_4, A) \to H^0(B, H^1(C, A).$$

As the orders of $B$ and $H^0(C, A)$ are coprime, $H^1(B, H^0(C, A)) = 0$ and we have

$$H^1(A_4, A) \subset H^0(B, H^1(C, A)).$$

But $H^1(C, A)$ and therefore $H^0(B, H^1(C, A))$ is annihilated by multiplication by $4 = \mathrm{ord}(C)$. So is then $H^1(A_4, A)$, and therefore its image under restriction to any subgroup of order 3 is zero, which proves the claim and Proposition 3.1.

This finishes the proof of Theorem 0.2.

*Remark.* — Conjecture 0.1 would imply that an analogue of this result would hold for all dimensions. The case of dimension 4 is very easy: indeed, for the quotient $Y$ we have $h^{3,0}(Y) = 0$ from the irreducibility of the representation of $G$ on the tangent space, and this contradicts $\chi(\mathcal{O}_Y) = 0$. As we see, the problem in question is purely group-theoretic: if a finite $G$ acts freely on a complex torus $A$, can the representaion of $G$ in the tangent space of $A$ be irreducible?

We hope to return to this question in a forthcoming paper.

Notice that there are many examples of smooth orientable quotients of a *real* torus by a freely acting finite group such that their second Betti number is equal to one. For instance, in dimension six one can take $G = (\mathbb{Z}/2\mathbb{Z})^{\oplus 3}$ on the generators $a, b, c$ which acts on the torus as follows:

$$a(x_1, x_2, x_3, x_4, x_5, x_6) = (x_1 + 1/2, x_2, -x_3, -x_4, -x_5, -x_6)$$

$$b(x_1, x_2, x_3, x_4, x_5, x_6) = (x_1, -x_2 + 1/2, -x_3, x_4, -x_5, -x_6)$$

$$c(x_1, x_2, x_3, x_4, x_5, x_6) = (-x_1, x_2, x_3 + 1/2, -x_4, -x_5 + 1/2, -x_6).$$

However, also in the real case we do not know any examples for which the action of the group on the tangent space to the torus is irreducible.

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