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A SPLITTING THEOREM FOR THE KUPKA COMPONENT OF A FOLIATION OF $\mathbf{CP}^n$, $n \geq 6$. ADDENDUM TO A PAPER BY O. CALVO-ANDRADE AND N. SOARES

by Edoardo BALLICO

A codimension one singular holomorphic foliation F of $\mathbf{CP}^n$ is given by $\omega \in H^0(\mathbf{CP}^n, \Omega(k))$ (for some k) with $\omega \neq 0$, ω not vanishing on a hypersurface. The Kupka subset $K(F) := \{P \in \mathbf{CP}^n : \omega(P) = 0, d\omega(P) \neq 0\}$ of the singular set $S(F) := \{P \in \mathbf{CP}^n : \omega(P) = 0\}$ of F has remarkable properties (e.g. if not empty it is a smooth submanifold of pure codimension 2 with strong stability properties with respect to deformations of F). For much more on this topic, see [GLM] and [CS]. Let $K \neq \emptyset$ be a Kupka component of F, *i.e.* ([CS]) a connected component of $K(F)$. It was proved in [CL] that if K is a complete intersection, then F has a meromorphic first integral. Motivated by this result in [CS] it was conjectured and proved in some cases that every Kupka component is a complete intersection. Here we prove the following result.

THEOREM. — *Let F be a codimension 1 singular holomorphic foliation of $\mathbf{CP}^n$, $n \geq 6$, induced by $\omega \in H^0(\mathbf{CP}^n, \Omega^1(k))$ and such that the codimension 2 component of the singular set of F consists of a single compact Kupka component K with* $\deg(K) \neq k^2/4$. *Then K is a complete intersection.*

Key words: Singular foliations – Codimension 1 foliations – Kupka component – Complete intersection – Unstable vector bundle – Rank 2 vector bundle – Splitting of a vector bundle – Meromorphic first integral.
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The proof of this result uses in an essential way the results proven in [CS] and [GML]. We consider this paper as an addendum to [CS] and we invite the reader to turn to [GML] and [CS] for background, motivations, several results used here, and so on. For the results used on vector bundles and codimension 2 submanifolds of $\mathbf{CP}^n$, see [OSS], [FL] and [CS].

Assume that F is induced by $\omega \in H^0(\mathbf{CP}^n, \Omega^1(k))$. Let N_K be the normal bundle of K in $\mathbf{CP}^n$. By [CS], Corollary 3.5, N_K is the restriction $E|K$ to K of a rank 2 vector bundle E on $\mathbf{CP}^n$. K is a complete intersection if and only if E is the direct sum of two line bundles ([OSS]). If $n \geq 6$ every line bundle on K is the restriction of a line bundle on $\mathbf{CP}^n$ (see [FL]). Hence, by a very nice result of Faltings ([F]) if $n \geq 6$ and N_K is the direct sum of two line bundles, K is a complete intersection. By [CS], Cor. 4.5 (2), we may assume $k > 0$. By [CS], Th. 3.4 (2) to prove our result we may distinguish two cases, according to the transversal type of K. First assume that the transversal type of K is given by $\eta = pxdy - qydy$ with p, q positive relatively prime integers. Look at [GML], Th. 2.3 and its proof at page 321 (in particular the two lines before eq. (2.6)) and use that K is simply connected if $n \geq 6$ ([FL], Cor. 6.3). The quoted result [GML], Th. 2.3, was the essential imput for the proof of [CS], Th. 3.4; then [CS], Th. 3.4, and the calculations in [CS], §4, on the applications of the Baum-Bott formulas to K gave the proof of [CS], Cor. 4.5. By [GML], page 321, N_K is in this case the direct sum of two line bundles. Hence our theorem is proved in this case. Now assume that the transversal type of K is given by $\eta = pxdy - qydy$ with $p = q = 1$. By [CS], Th. 4.2, we have $\deg(K) = k^2/4$. Hence our theorem is proved even in this case.

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