

JEREMY TEITELBAUM

Geometry of an étale covering of the p -adic upper half plane

Annales de l'institut Fourier, tome 40, n° 1 (1990), p. 69-78

http://www.numdam.org/item?id=AIF_1990__40_1_69_0

© Annales de l'institut Fourier, 1990, tous droits réservés.

L'accès aux archives de la revue « Annales de l'institut Fourier » (<http://annalif.ujf-grenoble.fr/>) implique l'accord avec les conditions générales d'utilisation (<http://www.numdam.org/legal.php>). Toute utilisation commerciale ou impression systématique est constitutive d'une infraction pénale. Toute copie ou impression de ce fichier doit contenir la présente mention de copyright.

NUMDAM

Article numérisé dans le cadre du programme
Numérisation de documents anciens mathématiques

<http://www.numdam.org/>

GEOMETRY OF AN ÉTALE COVERING OF THE p -ADIC UPPER HALF PLANE

by Jeremy TEITELBAUM (*)

Introduction.

In this paper we describe the rigid geometry of the first layer in the tower of coverings of the p -adic upper half plane obtained from the division points of the formal group constructed in [2]. This covering is accessible because it is abelian and in some sense "tame." Using our results, we are able to describe the stable special fiber at p of Shimura curves with a very small amount of level p structure.

Preliminaries.

Let $\hat{\mathcal{H}}_p$ denote the formal scheme over \mathbf{Z}_p constructed by Mumford ([4]) and commonly referred to as the p -adic upper half plane. Naively, $\hat{\mathcal{H}}_p$ is the complement of the \mathbf{Q}_p -rational points in \mathbf{P}^1 . We let \mathcal{H}_p be the rigid analytic space associated to $\hat{\mathcal{H}}_p$.

In [2], Drinfeld shows that $\hat{\mathcal{H}}_p$ is a parameter space for two-dimensional formal groups with a certain endomorphism structure. As a

(*) This research was supported by an NSF postdoctoral fellowship.

Key-words : p -adic uniformization – Shimura curves – Formal groups.
A.M.S. Classification : 11G18 – 11F85.

result, there is a universal family of formal groups \mathcal{G} over $\hat{\mathcal{H}}_p$. The subgroups obtained as the division points of this family of formal groups yield a tower of coverings of $\hat{\mathcal{H}}_p$. The rigid spaces associated to these coverings are a family of étale coverings of \mathcal{H}_p . Our goal in this work is to describe the simplest of these coverings.

For a detailed description of Drinfeld's universal formal group, we refer the reader to [8]. We recall here the basic definitions which we will require.

Let D be the quaternion division algebra over \mathbb{Q}_p , and let \mathcal{O}_D be the maximal order in D . A formal group G of dimension 2 and height 4 over a ring R on which p is nilpotent is called a special, formal \mathcal{O}_D -module (abbreviated *SFD*-module) provided that \mathcal{O}_D acts on G and, at each maximal ideal m of R , both characters of the residue field of \mathcal{O}_D occur in the tangent space to G at m . Over $\bar{\mathbb{F}}_p$, all *SFD*-modules are isogenous, so fix one such module Φ . With these conventions, we can state Drinfeld's theorem.

THEOREM (Drinfeld). — $\hat{\mathcal{H}}_p \otimes \mathrm{Spf} \hat{\mathbb{Z}}_p^{ur}$ (over \mathbb{Z}_p) represents the functor which assigns to a ring R on which p is nilpotent the set of isomorphism classes of triples (ψ, G, ρ) where

1. $\psi : \hat{\mathbb{Z}}_p^{ur}/p\hat{\mathbb{Z}}_p^{ur} \rightarrow R/p$ is a homomorphism,
2. G is an *SFD*-module over R ,
3. and $\rho : \psi_*\Phi \rightarrow G \otimes R/p$ is a "quasi-isogeny of height zero," which means that ρ is an isogeny with a certain normalization condition which will not be important in our work.

We let (Ψ, \mathcal{G}, Π) be the universal triple over $\hat{\mathcal{H}}_p \otimes \mathrm{Spf} \hat{\mathbb{Z}}_p^{ur}$, and \mathcal{G}_Π be the kernel of multiplication by Π on \mathcal{G} . This is a finite, flat group scheme of order p^2 over $\hat{\mathcal{H}}_p \otimes \mathrm{Spf} \hat{\mathbb{Z}}_p^{ur}$. If $\mathrm{PGL}_2^+(\mathbb{Q}_p)$ denotes the image in $\mathrm{PGL}_2(\mathbb{Q}_p)$ of the elements of $\mathrm{GL}_2(\mathbb{Q}_p)$ with determinants of even p -adic order, then the action of $\mathrm{PGL}_2^+(\mathbb{Q}_p)$ on $\hat{\mathcal{H}}_p$ extends to an action on the universal triple over $\hat{\mathcal{H}}_p \otimes \mathrm{Spf} \hat{\mathbb{Z}}_p^{ur}$, and therefore to \mathcal{G}_Π . Furthermore, the residue field $\mathcal{O}_D/\Pi = \mathbb{F}$ acts on \mathcal{G}_Π , and this action commutes with the action of $\mathrm{PGL}_2^+(\mathbb{Q}_p)$. In fact, $\mathrm{GL}_2(\mathbb{Q}_p)$ acts on the universal triple; see [2] p. 109 for details.

Let us fix a map $\psi_0 : \hat{\mathbb{Z}}_p^{ur} \rightarrow \hat{\mathbb{Z}}_p^{ur}$ and consider the fiber of the induced projection map $\hat{\mathcal{H}}_p \otimes \mathrm{Spf} \hat{\mathbb{Z}}_p^{ur} \rightarrow \mathrm{Spf} \hat{\mathbb{Z}}_p^{ur}$. The resulting formal scheme is isomorphic to $\hat{\mathcal{H}}_p \otimes \mathrm{Spf} \hat{\mathbb{Z}}_p^{ur}$ viewed as a formal scheme over $\hat{\mathbb{Z}}_p^{ur}$ with structure map ψ_0 . The geometry of the formal scheme obtained in this

way does not depend on the choice of ψ_0 , so for the remainder of this paper we will abuse notation, suppress reference to ψ_0 , and denote by $\hat{\mathcal{H}}_p$ and \mathcal{H}_p respectively the formal and rigid p -adic upper half planes over \mathbf{Z}_p (resp. \mathbf{Q}_p), base-changed up to $\hat{\mathbf{Z}}_p^{ur}$ (resp. $\hat{\mathbf{Q}}_p^{ur}$.) Similarly, we base change the covering \mathcal{G}_Π to obtain a finite flat group scheme of order p^2 over (our base-changed) $\hat{\mathcal{H}}_p$. We let $\hat{\Sigma}$ be the complement of the zero section in this group scheme, and Σ be the associated rigid space. This paper is devoted to describing the rigid geometry of Σ .

Classification of Σ as μ_{p^2-1} -torsor.

The action of the endomorphism ring of \mathcal{G} induces an action of \mathbf{F}^\times on the covering Σ , as we mentioned before. The two embeddings

$$\sigma_0, \sigma_1 : \mathbf{F} \rightarrow \hat{\mathbf{Z}}_p^{ur} / p\hat{\mathbf{Z}}_p^{ur}$$

induce two different actions of μ_{p^2-1} on our fixed *SFD*-module Φ over $\bar{\mathbf{F}}_p$, and therefore, via the universal isogeny P , two different actions on Σ . This allows us to view Σ as a μ_{p^2-1} torsor in two different ways.

DEFINITION 1. — *Let*

$$c_i(\Sigma) \in H_{\text{ét}}^1(\mathcal{H}_p \otimes \text{Spf } \mathbf{C}_p, \mu_{p^2-1})$$

be the class representing the covering $\Sigma \otimes \text{Spf } \mathbf{C}_p$ viewed as a μ_{p^2-1} torsor via the embedding

$$\tilde{\sigma}_i : W(\mathbf{F}) \rightarrow \hat{\mathbf{Z}}_p^{ur} \subset \mathbf{C}_p$$

induced by σ_i .

The following lemma relates the two classes.

LEMMA 2. — $c_i(\Sigma) = pc_{i+1}(\Sigma)$, *reading subscripts mod 2.*

Proof. — Changing the choice of σ_i twists the μ_{p^2-1} action on Σ by $\zeta \mapsto \zeta^p$. □

Our goal now is to determine the classes c_i precisely. Let \mathcal{T} be the tree of $\text{SL}_2(\mathbf{Q}_p)$. We fix a reduction map $r : \mathcal{H}_p \rightarrow \mathcal{T}$ which is compatible with the action of $\text{SL}_2(\mathbf{Q}_p)$. For one of many detailed descriptions of the relation between \mathcal{T} and \mathcal{H}_p , see [8], pp. 648–660.

The following theorem of Drinfeld ([1]) relates the cohomology of \mathcal{H}_p to the tree \mathcal{T} .

THEOREM 3 (Drinfeld [1]). — *If N is an integer prime to p , there is an isomorphism*

$$\partial : H_{et}^1(\mathcal{H}_p \otimes \mathrm{Spf} \mathbf{C}_p, \mu_N) \rightarrow C_{har}^1(\mathcal{T}, \mathbf{Z}/N\mathbf{Z})$$

where \mathcal{T} is the tree of SL_2 and $C_{har}^1(\mathcal{T}, \mathbf{Z}/N\mathbf{Z})$ is the group of harmonic 1-cochains on \mathcal{T} — that is, the set of functions f on the edges of \mathcal{T} such that, for each vertex v , f satisfies

$$\sum_{e \rightarrow v} f(e) = 0$$

where the sum is over the oriented edges of \mathcal{T} meeting v .

Let us briefly recall how the map ∂ of the theorem is constructed. Suppose Y is a torsor over $\mathcal{H}_p \otimes \mathrm{Spf} \mathbf{C}_p$. Let U be the admissible open set in \mathcal{H}_p corresponding to a vertex v in \mathcal{T} , together with its bounding edges. It follows from Lemma 2 of [8] that U is a $\mathrm{GL}_2(\mathbf{Q}_p)$ translate of the standard open set

$$V = \{P \in \mathcal{H}_p : 1/p < |z(P)| < p\} - \bigcup_{i=1}^{p-1} B_{1/p}^+(i)$$

where $B_r^+(i)$ denotes the closed ball centered at i of radius r . Therefore $\mathrm{Pic}(U) = 0$ and so

$$\mathcal{O}_Y = \mathcal{O}_U[T]/(T^{p^2-1} - f)$$

where μ_{p^2-1} acts by multiplication on T , and f is uniquely determined up to $(p^2 - 1)^{st}$ powers.

Let $e = \{v, v'\}$ be an (oriented) edge leaving v . To evaluate $\partial(Y)(e)$, choose a coordinate function z on U such that if $P \in U$ reduces to v then $|z(P)| = 1$ while if $P \in U$ reduces to e then

$$|p| < |z(P)| < 1.$$

Then we let

$$\partial(Y)(e) = \mathrm{Res}_e df/f \pmod{p^2 - 1}$$

where Res_e denotes the rigid analytic “annular residue” computed with respect to the selected coordinate z (which determines the sign of Res .)

Since $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ acts on Σ , the class $c_i(\Sigma)$ is invariant by $\mathrm{PGL}_2^+(\mathbf{Q}_p)$. As we see in the next lemma, this is a very strong condition.

LEMMA 4. — *Suppose Y is a $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ -invariant torsor over $\mathcal{H}_p \otimes \mathrm{Spf} \mathbf{C}_p$, and $\partial(Y)$ is the associated harmonic 1-cocycle. Then $\partial(Y)$ satisfies the following conditions:*

1. $\partial(Y)(e) = \partial(Y)(\gamma e)$ for all $\gamma \in \text{PGL}_2^+(\mathbf{Q}_p)$ and all oriented edges e of \mathcal{T} .

2. $\partial(Y)(e) \equiv 0 \pmod{p-1}$ on all edges e of \mathcal{T} .

Proof. — The first property is a consequence of the invariance of Y by $\text{PGL}_2^+(\mathbf{Q}_p)$ and the fact that $\text{PGL}_2^+(\mathbf{Q}_p)$ preserves the orientation of edges in \mathcal{T} . For the second, observe that all the edges e leaving any vertex v are permuted transitively by $\text{PGL}_2^+(\mathbf{Q}_p)$. Therefore $\partial(Y)(e)$ is a constant x on all edges e leaving v . From the harmonicity condition we obtain

$$\sum_{e \rightarrow v} \partial(Y)(e) = (p+1)x \equiv 0 \pmod{p^2-1}$$

since there are $p+1$ edges leaving v . This proves the lemma. □

In order to give a precise statement of our theorem, we must invoke the relationship between orientations on \mathcal{T} and the embeddings σ_i . As Drinfeld shows, and we explain in Lemma 14 of [8], the action of Π on the tangent space T to \mathcal{G} allows us to partition the vertices of \mathcal{T} into two classes labeled with the σ_i . To describe this partition, first decompose T into σ_i -eigenspaces for the action for the quadratic unramified extension of \mathbf{Z}_p inside \mathcal{O}_D . A vertex v is labelled with σ_i if $\Pi T^i \subset pT^{i+1}$ over the affinoid reducing to v where T^i is the σ_i -eigenspace in T . Vertices of the two classes alternate in the tree.

Since $\partial_i(\Sigma)$ is $\text{PGL}_2^+(\mathbf{Q}_p)$ -invariant, and this group permutes the edges of \mathcal{T} transitively, it suffices to specify the value of $\partial_i(\Sigma)$ on a single edge. This is the content of our theorem.

THEOREM 5. — *Let $e = [v, v']$ be an edge of \mathcal{T} . Suppose that v is labeled with σ_i . Then*

$$\partial_i(\Sigma)(e) = p - 1 .$$

Proof. — Notice first of all that, by Lemmas 2 and 4,

$$\begin{aligned} \partial_i(\Sigma)([v, v']) &= -\partial_i(\Sigma)([v', v]) \\ &= p\partial_i(\Sigma)([v', v]) \\ &= \partial_{i+1}(\Sigma)([v', v]) \end{aligned}$$

and therefore we may assume that v is labeled with σ_0 . Let U be the affine open set of $\hat{\mathcal{H}}_p$ corresponding to e . It follows from Lemma 2 of [8] that U is a $\text{PGL}_2^+(\mathbf{Q}_p)$ translate of the standard open set

$$U(1) = \{P \in \mathcal{H}_p : 1/p \leq |z(P)| \leq 1\} - \bigcup_{i=1}^{p-1} (B_1(i) \cup B_{1/p}(pi))$$

where $B_r(i)$ denotes the open disc of radius r centered at i . Therefore the coordinate ring of U is isomorphic to

$$\hat{R} = \varprojlim R/p^n R$$

where

$$R = \frac{\mathbf{Z}[z_0, z_1]}{(z_0 z_1 - p)} \left[\frac{1}{1 - z_0^{p-1}}, \frac{1}{1 - z_1^{p-1}} \right].$$

As we recalled prior to stating this theorem, the tangent space T to \mathcal{G} over U is free of rank 2, and it carries a grading coming from the action of the maximal unramified extension of \mathbf{Z}_p in \mathcal{O}_D . Write $T = \hat{R}t_0 \oplus \hat{R}t_1$. Referring to [8] p. 656, we see that the Π action on T is $\Pi t_0 = z_1 t_1$ and $\Pi t_1 = z_0 t_0$. With these conventions, the vertex v of \mathcal{T} labeled with σ_0 corresponds to the region where z_0 is a unit; the vertex v' labeled with σ_1 corresponds to the region where z_1 is a unit.

Let Ω be the cotangent space to \mathcal{G} . Then $\Omega_\Pi = \Omega/\Pi\Omega$ is naturally the cotangent space to \mathcal{G}_Π . If ω_0 and ω_1 generate the graded pieces of Ω , then we must have $\Pi\omega_i = z_i\omega_{i+1}$. It follows that

$$(1) \quad \Omega_\Pi = (\hat{R}/z_1\hat{R})\omega_0 \oplus (\hat{R}/z_0\hat{R})\omega_1.$$

The finite flat group scheme \mathcal{G}_Π , together with its action by \mathbf{F} is of the type classified by Raynaud. Applying his classification (see [5], Corollary 1.5.1) we see that the coordinate B ring of \mathcal{G}_Π must have the form

$$B = \hat{R}[X_0, X_1]/(X_0^p - \delta_0 X_1, X_1^p - \delta_1 X_0)$$

where the functions δ_i and p/δ_i belong to \hat{R}^\times . In addition, Raynaud shows that the natural identification of Ω_Π with I/I^2 (I being the augmentation ideal) means that

$$(2) \quad \Omega_\Pi = (\hat{R}/\delta_1\hat{R})X_0 \oplus (\hat{R}/\delta_0\hat{R})X_1.$$

Combining (1) and (2), we see that z_i and δ_i differ by a unit of \hat{R} .

We now have enough information to compute the class of Σ . Indeed, Σ is defined over U by the equation

$$X_0^{p^2-1} - \delta_0^p \delta_1 = 0.$$

The group \mathbf{F}^\times acts on X_0 through the embedding σ_0 . Let us write $f = \delta_0^p \delta_1$. Then

$$\partial_0(\Sigma)(e) = \text{Res}_{z_0} df/f \pmod{p^2 - 1}.$$

Our results above tell us that $\delta_0^p \delta_1 = pz_0^{p-1}h(z)$ where h is a unit in \hat{R} . This residue is clearly $p - 1$. \square

Thanks to this theorem, we can construct Σ over \mathbb{C}_p . Let X be the non-singular projective curve over \mathbb{C}_p defined by the affine equation

$$(3) \quad Y^{p+1} = z - z^p .$$

Let $W \subset X$ be the admissible open set of points P on X such that

$$(4) \quad |p^{1/(p+1)}| < |Y(P)| < |p^{-p/(p+1)}| .$$

COROLLARY 6. — *Over \mathbb{C}_p , Σ consists of $p-1$ isomorphic connected components. Each such component has a covering by admissible open sets isomorphic to W . The nerve of this covering is the tree \mathcal{T} . If W_1 and W_2 are two elements of the covering, and $E = W_1 \cap W_2 \subset W_1$, then E is one of the boundary annuli of W_1 .*

Proof. — Let U be the subset of $\mathcal{H}_p \otimes \text{Spf } \mathbb{C}_p$ consisting of one vertex (say, labeled with σ_0), and its bounding edges. Then by Theorem 5, Σ over U is obtained by extracting the $p^2 - 1$ root of a function with order congruent to $p-1 \pmod{p^2 - 1}$ on each bounding annulus. If z is a coordinate on U , then the function $f(z) = (z - z^p)^{p-1}$ clearly meets this condition. Thus Σ is defined over U by the equation

$$Y_0^{p^2-1} = (z_0 - z_0^p)^{p-1}$$

where z_0 is an appropriate parameter on U . Notice first that this equation factors, so that Σ consists of $p-1$ connected components, and is built up out of pieces of the curve in (3) as claimed. It is a simple matter to check that the subset of W satisfying the inequality (4) has genus $(p^2 - p)/2$. Thus the reduction of Σ consists of curves meeting in double points, like the reduction of \mathcal{H}_p , except that the rational curves which appear in the reduction of \mathcal{H}_p are replaced by the curves of equation (3).

With somewhat more care one can determine the equations for Σ over $\hat{\mathbf{Z}}_p^{ur}$, instead of just over \mathbb{C}_p . Examining the end of the proof of Theorem 5 we see that over $\hat{\mathbf{Z}}_p^{ur}$, we can take $\delta_0^p \delta_1 = p(z - z^p)^{p-1}$ and therefore Σ is defined over U by the equation

$$Y_0^{p^2-1} = p(z_0 - z_0^p)^{p-1} .$$

From this one can obtain a minimal regular model for Σ over $\hat{\mathbf{Z}}_p^{ur}$ by blowing-up. This is a straightforward computational problem whose solution we omit, although we do point out that the minimal model has no components of multiplicity one.

Finally, notice that the action of $\text{PGL}_2^+(\mathbb{Q}_p)$ on Σ extends to an action of $\text{PGL}_2(\mathbb{Q}_p)$. Indeed, choose $\tau \in \text{PGL}_2(\mathbb{Q}_p) - \text{PGL}_2^+(\mathbb{Q}_p)$. Then

$c_i(\tau^*\Sigma) = -c_i(\Sigma)$ since τ reverses orientations. It follows that $\tau^*\Sigma$ is isomorphic to Σ as a rigid space but that the action of μ_{p^2-1} on $\tau^*\Sigma$ is twisted by Frobenius. This could have been deduced, of course, from the general construction of Drinfeld.

Application to Shimura curves.

Now we examine the implications of Theorem 5 for the geometry of Shimura curves. Let Δ be an indefinite quaternion algebra over \mathbf{Q} ramified at p and let L be a maximal order in Δ . Suppose $n \geq 3$ is prime to p . Let S_n be the scheme representing the functor which associates to a scheme S the set of isomorphism classes of abelian surfaces over S with an L action and a level n -structure, and let S_n^{an} be the associated p -adic rigid space.

Let $\wp \subset L$ be the unique prime ideal above p . Let $S_{n,\wp}$ be the covering of S_n which classifies abelian L surfaces together with level n structure and level \wp structure. As before, $S_{n,\wp}^{\text{an}}$ is the associated p -adic rigid space.

Let

$$X_n = U_n \backslash (\Delta' \otimes \mathbf{A}^f)^\times / \Delta'^\times$$

where Δ' is the definite quaternion algebra obtained from Δ by interchange of invariants at p and ∞ . Let U_n be the principal congruence subgroup

$$U_n \subset \prod_{l \neq p} (L \otimes \mathbf{Z}_l).$$

With this notation, we can state (a slightly simplified form of) Drinfeld's theorem.

THEOREM (Drinfeld [2]). — *There are isomorphisms*

$$(5) \quad \text{GL}_2(\mathbf{Q}_p) \backslash \mathcal{H}_p \otimes \text{Spf } \hat{\mathbf{Z}}_p^{ur} \times X_n \rightarrow S_n^{\text{an}}$$

and

$$(6) \quad \text{GL}_2(\mathbf{Q}_p) \backslash \Sigma \times X_n \rightarrow S_{n,\wp}^{\text{an}}.$$

Furthermore, as Drinfeld points out, the quotient in (5) is actually the union of a finite number of components, each of the form

$$\Gamma \backslash \mathcal{H}_p \otimes \text{Spf } \hat{\mathbf{Z}}_p^{ur}$$

where Γ is a Schottky group.

Combining this with our geometric description of Σ , we obtain the following theorem.

THEOREM 8. — *Let $S_n(\Gamma) = \Gamma \backslash \mathcal{H}_p \otimes \text{Spf } \hat{\mathbf{Z}}_p^{ur}$ be one of the components of S_n^{an} . Suppose $\Phi = \mathcal{T}/\Gamma$ is the intersection graph of $S_n(\Gamma)$. Then the covering $S_{n,p}^{\text{an}}$ over $S_n(\Gamma)$ has a stable model over \mathbf{C}_p consisting of $p - 1$ components. The reduction of each such component has intersection graph Φ , but the vertices correspond to curves with the equation (3) rather than to rational curves.*

For the sake of concreteness, we supply an example. Suppose that Δ has discriminant 26 and that $p = 2$. Then Δ' has discriminant 13. Choose an embedding $\Delta' \otimes \mathbf{Z}_p \hookrightarrow M_2(\mathbf{Q}_p)$. Let A be a maximal $\mathbf{Z}[1/2]$ order in Δ' , and let

$$\Gamma = \{ \gamma \in A : nr(\gamma) = 2^k, k \text{ even} \} .$$

The Shimura curve S_1^{an} of level 1 (over \mathbf{C}_p) is the quotient

$$\Gamma \backslash \mathcal{H}_p \otimes \text{Spf } \mathbf{C}_p .$$

We are allowed to consider level 1 since Δ' has no multiplicative torsion. Since Δ' has class number 1, it is not hard to check that the special fiber of S_1 consists of two rational curves meeting in 3 points — see [3] or [6]. By the theorem, the special fiber of $S_{1,p}$ consists of two copies of the elliptic curve $Y^3 = z - z^2$ crossing in three points.

Conclusions.

In conclusion, we mention two questions related to our subject matter. The first, rather naturally, is to obtain information on the higher coverings of the p -adic upper half plane; and, in particular, on the covering obtained from the p -torsion on Drinfeld's formal group. This is clearly a much harder problem than the one we have solved, since the higher coverings are not abelian and are in some sense "wildly ramified."

From a rigid analytic point of view, however, it would also be interesting to study the class of curves which admit a uniformization by Σ . Such curves are a type of generalized Mumford curve, and it would be worthwhile to extend the p -adic analytic theory of Mumford curves to this more general setting. In particular, the Jacobians of these curves are semi-abelian schemes, and it would be interesting to obtain some form of the Manin-Drinfeld theory of p -adic automorphic forms on Σ .

BIBLIOGRAPHIE

- [1] V.G. DRINFELD, Elliptic modules, *Math. USSR Sbornik*, 23(4) (1976).
- [2] V.G. DRINFELD, Coverings of p -adic symmetric regions, *Functional Analysis and its Applications*, 10(2) (1976), 29–40.
- [3] A. KURIHARA, On some examples of equations defining Shimura curves and the Mumford uniformization, *J. Fac. Sci. Univ. Tokyo, Sec. IA*, 25 (1979).
- [4] D. MUMFORD, An analytic construction of degenerating curves over complete local rings, *Compos. Math.*, 24 (1972), 129–174.
- [5] M. RAYNAUD, Schémas en groupes de type (p, \dots, p) , *Bull. Soc. Math. France*, 102 (1974).
- [6] K. RIBET, Bimodules and abelian surfaces, Technical Report PAM-423, Center for Pure and Applied Mathematics, University of California, Berkeley, August 1988.
- [7] P. SCHNEIDER and U. STUHLER, The cohomology of p -adic symmetric spaces, preprint, 1988.
- [8] J. TEITELBAUM, On Drinfeld's universal formal group over the p -adic upper half plane, *Mathematische Annalen*, 284 (1989), 647–674.

Manuscrit reçu le 5 mai 1989,
révisé le 18 décembre 1989.

Jeremy TEITELBAUM,
Mathematics Department
University of Michigan
Ann Arbor, Michigan 48109 (U.S.A.).