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# ON THE LOCAL PSEUDOCONVEXITY OF CERTAIN ANALYTIC FAMILIES OF $\mathbb{C}$

**by Takeo OHSAWA**

*Dedicated to Jean-Pierre Demailly on his sixtieth birthday*

Abstract. — For a class of weakly 1-complete $\mathbb{C}$ bundles over compact Riemann surfaces, for which canonical plurisubharmonic exhaustion functions on the total spaces are known, some cases are described where such functions can be extended to a plurisubharmonic exhaustion function on analytic families of the $\mathbb{C}$ bundles. The nonextendable cases are also discussed.

Résumé. — Nous donnons des conditions pour que certaines fonctions analytiques plurisousharmoniques exhaustives sur des variétés faiblement 1-complètes qui sont des fibrés en droites affines au dessus de surfaces de Riemann soient extensibles à des familles analytiques de fonctions plurisousharmoniques exhaustives. Un exemple de famille non-extensible est également présenté.

## 1. Introduction

It is well known from the works of Oka [23, 24] that every domain of holomorphy over $\mathbb{C}^n$ is holomorphically convex (cf. [4]). This basic result is contained in an assertion, which is the main result of [24], that every locally pseudoconvex domain over $\mathbb{C}^n$ is holomorphically convex. The latter has been generalized in various situations on complex manifolds and on complex spaces with singularities (cf. [2, 11, 12, 26, 27, 28, 31]). In the present article, we shall say that a reduced complex space $X$ with a holomorphic map $\pi$ to a complex space $T$ is locally pseudoconvex over $T$, with a slight abuse of language, if every point of $T$ admits a neighborhood $U$ such that $\pi^{-1}(U)$ admits a $C^\infty$ plurisubharmonic exhaustion function. Following Nakano [17] we shall say that $X$ is weakly 1-complete if it admits a $C^\infty$ plurisubharmonic exhaustion function. This terminology comes

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from Grauert's characterization of Stein manifolds as manifolds admitting strictly plurisubharmonic exhaustion functions (cf. [12]), which are said to be 1-complete in [1]. It is known that a weakly 1-complete manifold with a positive line bundle is holomorphically embeddable into $\mathbb{CP}^n$ $(n \gg 1)$ (cf. [26]) and that weakly 1-complete manifolds with negative canonical bundles are holomorphically convex (cf. [27]).

We recall that a complex manifold is called $q$-complete if it carries a $C^\infty$ exhaustion function whose Levi form admits everywhere less than $q$ nonpositive eigenvalues. The notion of $q$-completeness is generalized to complex spaces without difficulty. It is known that every noncompact (and paracompact) complex space of dimension $n$ without compact irreducible components is $n$-complete (cf. [15] and [20]). It depends on the choices of $X$, $T$, $\pi$ and $q$ whether or not a given locally pseudoconvex space $X$ over $T$ is $q$-complete. Actually, in contrast to the case of Oka, where $\pi$ is a local homeomorphism, the local pseudoconvexity of $\pi : X \to T$ does not necessarily imply that $X$ is weakly 1-complete even if $T = \mathbb{C}^n$. Such a phenomenon has been observed in many situations with illustrative counterexamples (cf. [5, 9, 10, 25, 29]). On the other hand, results have been obtained concerning the weak 1-completeness of locally pseudoconvex subdomains in complex manifolds both in the positive and negative directions, motivated by remarks of Grauert (cf. [6, 8, 13, 14, 18, 19, 22]). In particular, Diederich and Fornaess showed in [6] that there exist locally pseudoconvex and smoothly bounded domains of dimension $n$ in compact manifolds which are not weakly 1-complete if $n \geqslant 3$. The purpose of the present article is to prove several affirmative results in this context as a continuation of [8] and [22], focusing on the notions of local pseudoconvexity and weak 1-completeness. The aim is to strengthen a basic fact that (the total space of) a holomorphic affine line bundle over a compact Riemann surface is weakly 1-complete if and only if its Chern class is nonpositive, which was first observed by Ueda [30] to the knowledge of the author. We shall prove its relative variant as a result of preliminary nature.

Theorem 1.1. — *Let $T$ be a complex manifold, let $p : S \to T$ be a proper holomorphic map with smooth fibers of dimension one, and let $q : L \to S$ be a holomorphic affine line bundle. Then $p \circ q : L \to T$ is locally pseudoconvex if one of the following conditions is satisfied.*

(1) *Fibers $L_t (t \in T)$ of $p \circ q$ are of negative degrees over the fibers $S_t$ of $p$.*
(2) *$L_t$ are topologically trivial over $S_t$ and not equivalent to holomorphic line bundles.*

(3) *$L \to S$ is $U(1)$-flat.*

We shall also show that there exists a holomorphic affine line bundle over the trivial family of an elliptic curve $(\mathbb{C}\setminus\{0\})/\mathbb{Z}$ over $\mathbb{C}$ which is not locally pseudoconvex over $\mathbb{C}$ (see Section 3).

The main case (2) of Theorem 1.1 has already been discussed in [22] when $L$ is flat. For this case, we had to employ Ueda's method in [30] taking the parameter dependence of harmonic sections of $L_t$ into account. The result was applied to prove the weak 1-completeness of holomorphic disc bundles over $S$ when $T$ is Stein. Here we shall be contented with local pseudoconvexity, since the situation becomes more delicate as the above mentioned counterexample shows.

## 2. Proof of Theorem 1.1

The proof of Theorem 1.1 is a slight modification of an argument employed in [22] for the proof of the weak 1-completeness of the disc bundles over analytic families of compact Riemann surfaces over Stein manifolds. Hence the materials below are mostly contained in [22]. However the rest is somewhat delicate, so that it is not only for the convenience of the reader that they are recalled here. First we recall basic facts on analytic affine line bundles over complex manifolds. By definition, a holomorphic affine line bundle over a complex manifold $M$ is a holomorphic fiber bundle $q : X \to M$ whose fibers are equivalent to $\mathbb{C}$. Note that $\operatorname{Aut}\mathbb{C} = \{az+b; a,b \in \mathbb{C}, a \neq 0\}$, where $z$ denotes the coordinate of $\mathbb{C}$. So, affine line bundles are simply the complex plane with holomorphic parameters. For the pseudoconvexity property of $X$, it is easy to see that $X$ is Stein if $M$ is Stein. Note that the converse is not true. Indeed, for a holomorphic line bundle of degree less than 1 over the Riemann sphere, say $\pi : L \to \hat{\mathbb{C}}(:= \mathbb{C}\cup\{\infty\})$, any affine line bundle over $\hat{\mathbb{C}}$ associated to a nonzero element of $H^{0,1}(\hat{\mathbb{C}}, L)$ is Stein, where $H^{p,q}(M,E)$ generally denotes the Dolbeault cohomology groups of type $(p,q)$ for a holomorphic vector bundle $E$ over $M$. Namely, let $L$ be given by patching $\mathbb{C}\times\mathbb{C} = \{(z,\zeta); z,\zeta \in \mathbb{C}\}$ and $\mathbb{C}\times\mathbb{C} = \{(w,\xi); w,\xi \in \mathbb{C}\}$ by the map $w = \frac{1}{z}, \xi = z^m\zeta$ for $m \geqslant 2$ and let $X \to \hat{\mathbb{C}}$ be the bundle defined by $(z,\zeta) \mapsto (\frac{1}{z}, z^m\zeta + z^{m-1})$, whose associated line bundle is $L$.

Then the functions $z^m\zeta + z^{m-1}$ and $z^{m-1}\zeta + z^{m-2}$ are holomorphically extendable to $X$, from which it is easy to see that $X$ is Stein. More remarkable fact is that, for any compact Riemann surface $R$ and a holomorphic line bundle $F \to R$ with $\deg F \leqslant 0$, any affine line bundle associated to a nonzero element of $H^{0,1}(R, F)$ is Stein. We shall recall the construction of a plurisubharmonic function in [8] for the case $\deg F = 0$, because this method has to be employed with an additional care later.

Theorem 2.1. — *Topologically trivial holomorphic affine line bundles over compact Kähler manifolds are weakly 1-complete.*

*Proof.* — Let $M$ be a compact Kähler manifold, let $\pi : L \to M$ be a topologically trivial holomorphic affine line bundle, and let $L_0$ be the holomorphic line bundle associated to $L$. Then, since $M$ is Kähler, one can find an open covering $\{U_\alpha\}_{\alpha \in A}$ of $M$ and local trivializations $\varphi_\alpha : \pi^{-1}(U_\alpha) \cong U_\alpha \times \mathbb{C} = \{(z, \zeta_\alpha); z \in U_\alpha, \zeta_\alpha \in \mathbb{C}\}$ in such a way that $\varphi_\alpha \circ \varphi_\beta^{-1}(z, \zeta_\beta) = (z, e^{i\theta_{\alpha\beta}}\zeta_\beta + a_{\alpha\beta}(z))$ holds on $\varphi_\beta(\pi^{-1}(U_\alpha \cap U_\beta))$ for some $\theta_{\alpha\beta} \in \mathbb{R}$ and $a_{\alpha\beta} \in \mathcal{O}(U_\alpha \cap U_\beta)$, for all $\alpha$ and $\beta$ in $A$, where $\mathcal{O}(U_\alpha \cap U_\beta)$ denotes the set of holomorphic functions on $U_\alpha \cap U_\beta$. This is a basic fact in Kähler geometry relying on the $\partial\bar\partial$-lemma. Note that $\{e^{i\theta_{\alpha\beta}}\}$ is a system of transition functions for $L_0$ and $\{a_{\alpha\beta}\}$ is a representative of an element of $H^{0,1}(M, L_0)$ as an $L_0$-valued 1-cocycle. Then, applying the Kähler condition again, by replacing $\{U_\alpha\}$ by its refinement if necessary, one can find $a_\alpha, b_\alpha \in \mathcal{O}(U_\alpha)$ such that $a_{\alpha\beta} = a_\alpha + b_\alpha e^{i\theta_{\alpha\beta}}(a_\beta + b_\beta)$ holds on $U_\alpha \cap U_\beta$. Recall that this is also a consequence of the $\partial\bar\partial - lemma$ (cf. [7, Lemma 2]). For simplicity we put $h_\alpha = a_\alpha + b_\alpha$. The system $h_\alpha$ is narutally identified with a plurisubharmonic section of the bundle $L \to M$. Then it is straightforward that the function $\Phi = |\zeta_\alpha - h_\alpha|^2$ is a well-defined plurisubharmonic exhaustion function on $L$. Indeed, the well-definedness and the exhaustiveness are obvious, and the plurisubharmonicity is immediate from $\partial\bar\partial\Phi = d\zeta_\alpha d\zeta_\alpha - d\zeta_\alpha\bar\partial h_\alpha - d\zeta_\alpha\partial h_\alpha + \partial h_\alpha\bar\partial h_\alpha + \partial h_\alpha\bar\partial h_\alpha \geqslant \partial h_\alpha\bar\partial h_\alpha$, where $\partial\bar\partial\Phi$ is identified with the complex Hessian of $\Phi$. □

*Proof of Theorem 1.1.* — Let the notation be as in the statement. Since the assertion is local in $T$, we may assume that $T$ is a polydisc, say $T = \{t = (t_1, \ldots, t_m) \in \mathbb{C}^m; |t| := \max|t_j| < 1\}$. We put $T_r = \{t \in T; |t| < r\}$. Let us denote $(0, \ldots, 0) \in \mathbb{C}^m$ simply by 0, let $S_t = p^{-1}(t)$, and let $L_t = q^{-1}(S_t)$. Let us first assume that the degree of the bundle $L_0 \to S_0$ is zero. Then we choose an open covering $\mathcal{U} = \{U_\alpha\}_{\alpha \in A}$ of $S$ and a system of local trivializations of $L$ associated to $\mathcal{U}$ so that the transition functions of $L_0 \to S_0$ with respect to $\{U_\alpha \cap S_0\}$ are of the form $\zeta_\alpha = e^{i\theta_{\alpha\beta}}\zeta_\beta + a_{\alpha\beta}(\theta_{\alpha\beta} \in \mathbb{R}, a_{\alpha\beta} \in \mathcal{O}(U_\alpha \cap U_\beta))$. As in the proof of Theorem 2.1, $\mathcal{U}$ can be chosen so that one can find harmonic functions $h_\alpha$ on $U_\alpha \cap S$ satisfying $a_{\alpha\beta} = h_\alpha - e^{i\theta_{\alpha\beta}}h_\beta$ on $U_\alpha \cap U_\beta \cap S_0$. Then, by virtue of the implicit function theorem, there exists $r > 0$ such that, by replacing $\mathcal{U}$ by its refinement if necessary, one can find $C^\infty$ extensions $\tilde{\theta}_{\alpha\beta}$ and $\tilde{h}_\alpha$ of $\theta_{\alpha\beta}$ and $h_\alpha$ to $U_\alpha \cap U_\beta \cap p^{-1}(T_r)$ and $U_\alpha \cap p^{-1}(T_r)$, respectively, in such a way that $\tilde{\theta}_{\alpha\beta}$ are real valued, $\tilde{h}_\alpha$ are harmonic on $S_t \cap U_\alpha$, and that the transition functions of $(p \circ q)^{-1}(T_r)$ are of the form $\zeta_\alpha = e^{i\tilde{\theta}_{\alpha\beta}}\zeta_\beta + \tilde{a}_{\alpha\beta}$, where $\tilde{a}_{\alpha\beta}|_{U_\alpha\cap U_\beta\cap S_t} \in \mathcal{O}(S_t \cap U_\alpha \cap U_\beta)$ and $\tilde{a}_{\alpha\beta} = \tilde{h_\alpha} e^{i\tilde{\theta}_{\alpha\beta}}\tilde{h}_\beta$ hold on $U_\alpha \cap U_\beta \cap p^{-1}(T_r)$. Let us put $\Phi = |\zeta_\alpha - \tilde{h}_\alpha|^2$. Then, as in the proof of Theorem 2.1, the restriction of $\Phi$ to $L_t$ is a plurisubharmonic exhaustion function for all $t \in T_r$. Suppose now that the condition (2) is satisfied. Then it is easy to see that $\Phi|_{L_t}$ is strictly plurisubharmonic on a dense subset of $L_t$. More explicitly, $\Phi|_{L_t\cap U_\alpha}$ is strictly plurisubharmonic on the complement of $\{(z,\zeta_\alpha) \in (U_\alpha \cap L_t) \times \mathbb{C}; \bar{\partial} h_\alpha(z) = 0\}$. Therefore, by shrinking $T_r$ if necessary, one can find a bounded $C^\infty$ function $\Psi$ on $(p \circ q)^{-1}(T_r)$ such that $(\Phi + \Psi)|_{L_t}$ is strictly plurisubharmonic for all $t \in T_r$. Then it is easy to verify that there exist $\epsilon > 0$ and $C > 0$ such that $\Phi + \Psi + C\Phi\|t\|^2$ is strictly plurisubharmonic on $(p \circ q)^{-1}(T_\epsilon)$. Here we put $\|t\|^2 = \sum_{j=1}^m |t_j|^2$. It is now obvious that $(p \circ q)^{-1}(T\epsilon)$ is 1-completefor sufficiently small $\epsilon$. If (1) is satisfied, the conclusion is obvious because the section at infinity of $L_0 \to S_0$ is a divisor with positive normal bundle and hence the section at infinity of $L|_{p^{-1}(T_\epsilon)} (0 < \epsilon \ll 1)$ has positive normal bundle, too. If (3) is satisfied, then the square of the euclidean distance along the fibers of $L$ is a well defined plurisubharmonic function which is exhaustive on each $L_t$. Hence $(p \circ q)^{-1}(V)$ is weakly 1-complete for any Stein open set $V \subset T$. □

## 3. A counterexample

Let $A$ be a complex torus of dimension one, say $A = (\mathbb{C} \setminus \{0\})/\mathbb{Z}$, where the action of $\mathbb{Z}$ on $\mathbb{C} \setminus \{0\}$ is given by $z \mapsto e^m z$ for $m \in \mathbb{Z}$. Over the product space $A \times \mathbb{C}$ as an analytic family of compact Riemann surfaces over $\mathbb{C}$, we define an affine line bundle $F \to A \times \mathbb{C}$ as the quotient of the trivial bundle $((\mathbb{C} \setminus \{0\}) \times \mathbb{C}) \times \mathbb{C} \to (\mathbb{C} \setminus \{0\}) \times \mathbb{C}$ by the action of $\mathbb{Z}$ defined by $(z, t, \zeta) \mapsto (e^m z, t, \zeta + mt)$. Had $F$ been locally pseudoconvex over $\mathbb{C}$, with respect to the map $\pi : F \to \mathbb{C}$ induced from the projection to the second factor, $\pi^{-1}(V)$ would be holomorphically convex for some neighborhood $V \ni 0$. Indeed, since the fibers of $\pi$ are all holomorphically convex and the canonical bundle of $F$ is trivial, holomorphic functions on $\pi^{-1}(t)$ would be holomorphically extendable to $\pi^{-1}(V)$ by an $L^2$ extension theorem if $\pi^{-1}(V)$ were weakly 1-complete (cf. [21]). This contradicts an obvious fact that $\pi^{-1}(0)$ cannot be blown down to $\mathbb{C}$ in $F$.

*Remark 3.1.* — Our example is similar in spirit to Demaillys example of non-Stein $\mathbb{C}^2$-bundle over $\mathbb{C} \setminus \{0\}$ in [5]. We recall that this bundle is defined as the quotient of $\mathbb{C} \times \mathbb{C}^2$ by the infinite cyclic group generated by $(\zeta, z, w) \mapsto (\zeta + 2\pi i, z^k - w, z), k \geqslant 2$. However, a big difference is that our family is Stein over $\mathbb{C} \setminus \{0\}$ because it is equivalent to $(\mathbb{C} \setminus \{0\})^3$ by the map induced from $(z, \zeta, t) \mapsto (ze^{-\frac{\zeta}{t}}, e^{\frac{2\pi i\zeta}{t}}, t)$.

*Remark 3.2.* — In contrast to our example, it was shown by Miebach [16] that the quotients of bounded homogeneous domains in $\mathbb{C}^n$ by $\mathbb{Z}$ are all Stein.

## 4. A note on families of $\mathbb{D}$

As was mentioned in the introduction, we have shown the following in [22].

Theorem 4.1. — *Let $p : S \to T$ be as in Theorem 1.1 and let $\pi : \mathcal{D} \to S$ be a holomorphic $\mathbb{D}$-bundle, where $\mathbb{D} = \{z \in \mathbb{C}; |z| < 1\}$. Then $\mathcal{D}$ is weakly 1-complete if $T$ is Stein.*

An open question is whether this remains true if $S$ is replaced by an analytic family of compact Kähler manifolds. We observe here that Theorem 1.1 is related to a very partial answer to this conjecture. To state it, let $p : Y \to T$ be a proper and smooth holomorphic map and let $q : X \to Y$ be a holomorphic disc bundle. Let $\chi : \tilde{Y} \to Y$ be the universal covering. Then $X$ is biholomorphically equivalent to the quotient of $\tilde{Y} \times \mathbb{D}$ by the $\pi_1(Y)$-action of the form $\gamma(y, \zeta) = (D(\gamma)y, \rho(\gamma)\zeta)(\gamma \in \pi_1(Y))$. Here $D$ denotes the canonical isomorphism between $\pi_1(Y)$ and the group of covering transformations of $\tilde{Y} \to Y$ and $\rho$ is a homomorphism from $\pi_1(Y)$ to $\operatorname{Aut} \mathbb{D}$. Let $\Gamma$ denote the image of $\rho$. In this situation, $X$ becomes holomorphically convex if $\Gamma$ is discrete and not cocompact, in virtue of a theorem of Behnke and Stein (cf. [3]). On the other hand,similarly as in the proof of the case (3) of Theorem 1.1, it is easy to see the following.

Theorem 4.2. — *$X$ is weakly 1-complete if $T$ is Stein and $\Gamma$ is commutative.*

Corollary 4.3. — *Let $L \to T$ be an analytic family of complex tori over a Stein manifold $T$ and let $X \to Y$ be a holomorphic disc bundle. Then $X$ is weakly 1-complete.*

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