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OVERCONVERGENT COHOMOLOGY OF HILBERT MODULAR VARIETIES AND p-ADIC L-FUNCTIONS

by Daniel BARRERA SALAZAR (*)

Abstract. — For each Hilbert modular form of *non-critical slope* we construct a p-adic distribution on the Galois group of the maximal abelian extension unramified outside p and ∞ of the totally real field. We prove that the distribution is admissible and interpolates the critical values of the complex L-function of the form. This construction is based on the study of the overconvergent cohomology of Hilbert modular varieties and certain cycles on these varieties.

Résumé. — Pour une forme de Hilbert de *pente non critique*, l'on construit une distribution p-adique sur le groupe de Galois de l'extension abélienne maximale du corps totalement réel, non-ramifiée en dehors de p et ∞. On démontre que la distribution obtenue est admissible et interpole les valeurs critiques de la fonction L complexe de la forme de Hilbert. Cette construction est basée sur l'étude de la cohomologie surconvergente des variétés modulaires de Hilbert et de certains cycles sur ces variétés.

1. Introduction

The construction and study of p-adic analytic L-functions for elliptic modular forms has been extensively studied by several authors using different approaches. In [14] the authors described the modular symbols approach and stated a conjecture about the exceptional zeros of those p-adic L-functions. Glenn Stevens gave a new construction of these p-adic

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L-functions using his theory of *overconvergent modular symbols* (see [18, 20]). His construction works also in families which allowed him to prove the exceptional zero conjecture (see [19, 21]). For Hilbert modular forms, the construction and study of p-adic analytic L-functions has been considered by authors such as Manin [13], Dabrowski and Panchishkin (see [8, 17]), Mok [15] and Dimitrov [9]. The construction in [9] is based on modular symbols setting a framework for generalising Stevens' work which is the object of this paper. Before stating our result, we first briefly recall Stevens' construction. Let p be a prime and $\mathbf{inc}_p : \overline{\mathbb{Q}} \hookrightarrow \overline{\mathbb{Q}}_p$ an embedding. Let $N \geqslant 4$ be an integer such that $(N, p) = 1$ and we put $\Gamma = \Gamma_0(pN)$. For $k \geqslant 2$ an integer and L a p-adic field, we denote by $\mathcal{D}_k(L)$ the space of L-valued locally analytic distributions on $\mathbb{Z}_p$ endowed with an action of Γ depending on k; the space of overconvergent modular symbols can then be described as $H^1_c(\Gamma, \mathcal{D}_k(L))$. Let f be a p-stabilization of a newform of weight k and level N, such that $U_p f = \alpha f$ and the p-adic valuation of α is strictly less than $k-1$. Using the Eichler–Shimura isomorphism one obtains a class $\varphi \in H^1_c(\Gamma, \mathrm{Sym}^{k-2}(L))$ such that $U_p\varphi = \alpha\varphi$. The first step in Steven's method is to lift φ to an element $\Phi \in H^1_c(\Gamma, \mathcal{D}_k(L))$ such that $U_p\Phi = \alpha\Phi$ (see [18]). This is an analogue of Coleman's classicality theorem for overconvergent modular forms. The p-adic L-function of f is then obtained by evaluating Φ on the cycle $\{\infty\} - \{0\}$.

Let now F be a totally real field of degree d. Consider a Hilbert modular variety Y_K of level $K \subset GL_2(\mathbb{A}_F^{(\infty)})$ and fix a cohomological weight λ of GL_2/F. Let L be a sufficiently large p-adic field. We denote by $\mathcal{D}_\lambda(L)$ the space of locally analytic distributions on $\mathcal{O}_F \otimes \mathbb{Z}_p$ with values in L, endowed with an action of a semigroup in $GL_2(\mathbb{Q}_p \otimes F)$ and having $\mathbb{V}_\lambda^\vee(L)$, the algebraic representation of weight λ, as a quotient. We consider the *overconvergent cohomology* $H^d_c(Y_K, \mathcal{D}_\lambda(L))$. This cohomology was introduced in [2] and [22], and is the natural object which generalises the overconvergent modular symbols. For a positive rational number $h \in \mathbb{Q}$ we are interested in the "slope-$\leqslant h$ part" of this cohomology, which essentially is the subspace of $H^d_c(Y_K, \mathcal{D}_\lambda(L))$ such that every eigenvalue of U_p has p-adic valuation $\leqslant h$ and is denoted by $H^d_c(Y_K, \mathcal{D}_\lambda(L))^{\leqslant h}$. This subspace has good properties when $H^d_c(Y_K, \mathcal{D}_\lambda(L))$ admits the so called "$\leqslant h$-slope decomposition" with respect to U_p, this property implies for example that $H^d_c(Y_K, \mathcal{D}_\lambda(L))^{\leqslant h}$ is a direct summand of $H^d_c(Y_K, \mathcal{D}_\lambda(L))$. The following theorem generalises Stevens' classicity theorem to the case of Hilbert modular forms (see 5.1 for more details). Its proof adapts the method of [22], where the analogous statement is established for the usual cohomology, namely by working on the boundary of the Borel–Serre compactification of Y_K.

THEOREM 1.1. — *$H^d_c(Y_K, \mathcal{D}_\lambda(L))$ admits a decomposition with respect to U_p. Moreover there exists $h(\lambda) > 0$ depending only on λ, such that if $h < h(\lambda)$ then we have a canonical isomorphism:*

$$H^d_c(Y_K, \mathcal{D}_\lambda(L))^{\leqslant h} \xrightarrow{\sim} H^d_c(Y_K, \mathbb{V}^\vee_\lambda(L))^{\leqslant h}.$$

An immediate consequence of this result is that given any cuspidal automorphic representation π of GL_2/F contributing to $H^d_c(Y_K, \mathbb{V}^\vee_\lambda(\mathbb{C}))$ and any p-stabilized new vector $\mathbf{f}$ in π which has non-critical slope, one has a well defined class $\Phi \in H^d_c(Y_K, \mathcal{D}_\lambda(L))$ (see 7.1). The main objective of this article is to attach a p-adic L-function to such a class. To achieve this we *evaluate* this class on the *automorphic cycles* introduced in [9]. Those cycles are morphisms of real analytic varieties $C_n : X_n \to Y_K$ where $X_n = \coprod_{\mathrm{Cl}^+_F(p^n)} (\mathbb{R}/\mathbb{Z})^{d-1} \times \mathbb{R}^\times_{\geqslant 0}$ and $\mathrm{Cl}^+_F(p^n)$ is the narrow ray class group of F. We remark that in the case $F = \mathbb{Q}$ we are considering the disjoint union of the paths joining a/p^n and ∞, for $a \in \{0, \dots, p^n - 1\}$ coprime to p, inside the modular curve. In 6.2 we use these cycles to define a distribution valued sequence of evaluations, ev_n for each $n \in \mathbb{N}$, on the overconvergent cohomology, which are analogues of the evaluations described in [9, §1.5]. Using ev_1 we construct a morphism:

$$H^d_c(Y_K, \mathcal{D}_\lambda(L)) \longrightarrow \mathcal{D}(\mathrm{Gal}_p, L), \tag{1.1}$$

where $\mathrm{Gal}_p = \mathrm{Gal}(F^{p,\infty}/F)$ where $F^{p,\infty}$ is the maximal abelian extension of F unramified outside p and ∞, and $\mathcal{D}(\mathrm{Gal}_p, L)$ is the space of locally analytic distributions on Gal_p. Then $\mu_{\mathbf{f}} \in \mathcal{D}(\mathrm{Gal}_p, L)$ is defined as the image of Φ under the map (1.1). Remark that $F^{p,\infty}$ contains the cyclotomic extension of F, and denote by $N : \mathrm{Gal}_p \to L^*$ the continuous character given by the cyclotomic character. For $s \in \mathbb{Z}_p$ and any continuous character $\chi : \mathrm{Gal}_p \to L^*$ we put:

$$L_p(\mathbf{f}, \chi, s) := \mu_{\mathbf{f}}(\chi N^{s-1}).$$

By construction this function is analytic in the variable $s \in \mathbb{Z}_p$. We are now in a position to state our main theorem:

Theorem 1.2. — *The distribution $\mu_{\mathbf{f}} \in \mathcal{D}(\mathrm{Gal}_p, L)$ is admissible. Let $\chi : \mathrm{Gal}_p \to L^\times$ be a finite order character of F such that $\chi_\sigma(-1) = 1$ for each $\sigma \in \Sigma_F$, then we have:*

$$L_p(\mathbf{f}, \chi, 1) = \mathbf{inc}_p \left(\frac{L^p(\pi \otimes \chi, 1)\tau(\chi)}{\Omega_\pi} \right) \prod_{\mathfrak{p}|p} Z_{\mathfrak{p}},$$

here $L^p(\pi \otimes \chi, s)$ is the L-function of π twisted by χ without the Euler factor in p, $\tau(\chi)$ is the Gauss sum, Ω_π is a period attached to π and $Z_{\mathfrak{p}}$ are local factors defined in terms of $\pi_{\mathfrak{p}}$ and $\chi_{\mathfrak{p}}$.

The proof of the admissibility uses crucially *all* evaluations ev_n. The proof of the interpolation formula is based on some computations given in [9].

As mentioned, a study of the p-adic properties of special values of L-functions attached to Hilbert modular forms was carried out for example in [8]. The present work has several advantages. For instance while in [8] the construction of the p-adic L-function was donne by using the Ranking method we use the theory of overconvergent modular symbols, which is more flexible to applications. In this direction, in [3], in collaboration with M. Dimitrov and A. Jorza, we extend the method of the present paper to construct p-adic L-functions in families, and investigate the exceptional zero conjecture for central critical values of Hilbert modular forms of any weight. We would like to point out that the Stevens' method for the construction of p-adic L-functions, has been developed in the context of GL_2 in [4, 24] and [5]. Finally we would like to mention that Januszewski constructed p-adic L-functions for $GL_n \times GL_{n-1}$ in [11]. We hope that these results will motivate similar constructions of p-adic L-functions for more general reductive groups. The article is structured as follows. In Section 2 we introduce some basic notations used in this work. In Section 3 we prove the existence of slope decomposition for the compactly supported cohomology of Hilbert modular varieties. In Section 4 we introduce some spaces of distributions. Section 5 is devoted to the proof of Theorem 1.1. In Section 6 we use automorphic cycles to construct evaluations on the overconvergent cohomology and we construct in particular the map (1.1). Finally in Section 7, we construct p-adic L-functions and we prove Theorem 1.2.

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2. Hilbert modular varieties

2.1. Notations

Let F be a totally real number field, $d = [F : \mathbb{Q}]$ and Σ_F be the set of embeddings of F in $\mathbb{C}$. In this paper we consider $\overline{\mathbb{Q}}$ as a subfield of $\mathbb{C}$ and then we identify Σ_F with $\mathrm{Hom}(F, \overline{\mathbb{Q}})$. We denote $\mathbf{t} = (1, \dots, 1) \in \mathbb{Z}^{\Sigma_F}$. We denote by $\mathbb{A}$ the ring of adeles over $\mathbb{Q}$ and $\mathbb{A}_f$ the ring of finite adeles. We put $\mathbb{A}_F = \mathbb{A} \otimes_{\mathbb{Q}} F$ and $\mathbb{A}_{F,f} = \mathbb{A}_f \otimes_{\mathbb{Q}} F$. Let $p \geqslant 2$ be a prime number and $\mathbf{inc}_p : \overline{\mathbb{Q}} \hookrightarrow \overline{\mathbb{Q}}_p$ an embedding. For each $\sigma \in \Sigma_F$ there is an unique $\mathfrak{p} \mid p$ in F such that $\mathbf{inc}_p \circ \sigma$ correspond to $\mathfrak{p}$. We obtain a decomposition $\Sigma_F = \bigsqcup_{\mathfrak{p}|p} \Sigma_{\mathfrak{p}}$, where $\Sigma_{\mathfrak{p}}$ is the set of $\sigma \in \Sigma_F$ corresponding to $\mathfrak{p}$ under $\mathbf{inc}_p$. Moreover, let $v_p : \overline{\mathbb{Q}}_p^{\times} \to \mathbb{Q}$ be the non-archimedean valuation such that $v_p(p) = 1$.

For each prime ideal $\mathfrak{p}$ over p we choose an uniformizer, $\varpi_{\mathfrak{p}}$, of $F_{\mathfrak{p}}$. In all this paper we suppose that $\varpi_{\mathfrak{p}}^{e_{\mathfrak{p}}} = p$, where $e_{\mathfrak{p}}$ is the inertia degree at $\mathfrak{p}$. Using the decomposition $F \otimes \mathbb{Q}_p = \prod_{\mathfrak{p}|p} F_{\mathfrak{p}}$ we obtain a group homomorphism $u : (F \otimes \mathbb{Q}_p)^{\times} \to (\mathcal{O}_F \otimes \mathbb{Z}_p)^{\times}$. Let $G = \mathrm{Res}_{\mathcal{O}_F/\mathbb{Z}} GL_2$, B be the Borel subgroup of the upper triangular matrices and T be the standard torus. Let Z be the center of G and we denote $G^{\mathrm{ad}} = G/Z$.

2.2. Hilbert modular varieties

2.2.1.

Let G_∞^+ be the connected component of the identity in $G(\mathbb{R})$, $Z_\infty = Z(\mathbb{R})$ and $K_\infty^+ = SO_2(F_\infty)$, where $F_\infty = F \otimes \mathbb{R}$. Let K be an open compact subgroup of $G(\mathbb{A}_f)$ then we define the *Hilbert modular variety of level K* by:

$$Y_K = G(\mathbb{Q}) \setminus G(\mathbb{A})/KK_\infty^+ Z_\infty.$$

This variety is a complex manifold and we can describe more explicitly its connected components. Let $\mathcal{C}_K^+ := F^\times \setminus \mathbb{A}_F^\times / \det(K) F_\infty^+$ and for each $\mathbf{x} \in \mathcal{C}_K^+$ we choose $g_{\mathbf{x}} \in G(\mathbb{A}_f)$ such that the image of $det(g_{\mathbf{y}})$ in $\mathcal{C}_K^+$ is $\mathbf{x}$. Then we have a decomposition:

$$Y_K = \bigsqcup_{\mathbf{x} \in \mathcal{C}_K^+} Y_{\mathbf{x}},$$

where $Y_{\mathbf{x}} = G(\mathbb{Q}) \setminus G(\mathbb{Q})g_{\mathbf{y}}KG_\infty^+/KK_\infty^+Z_\infty$. Moreover if we denote $\Gamma_{\mathbf{x}} = G(\mathbb{Q}) \cap g_{\mathbf{y}}Kg_{\mathbf{y}}^{-1}G_\infty^+$ then $Y_{\mathbf{x}} \simeq \Gamma_{\mathbf{x}} \setminus \mathbb{H}_F$, here $\mathbb{H}_F := \mathbb{H}^{\Sigma_F}$ and $\mathbb{H}$ is the upper half plane.

Hypothesis 2.1. — *We suppose in all this paper that for each $\mathbf{x} \in \mathcal{C}_K^+$ the group $\overline{\Gamma}_{\mathbf{x}} := \Gamma_{\mathbf{x}}/\Gamma_{\mathbf{x}} \cap Z(\mathbb{Q})$ is torsion-free.*

This hypothesis is satisfied if K is sufficiently small. In that case we deduce that Y_K is smooth and the fundamental group of $Y_{\mathbf{x}}$ is $\overline{\Gamma}_{\mathbf{x}}$.

Remark 2.2. — We will fix in all this work representatives $g_{\mathbf{x}}$ whose image in $G(\mathbb{Q}_p)$ is trivial.

2.2.2. Borel–Serre compactificacion

The variety Y_K is not compact, here we will describe the Borel–Serre compactificacion of Y_K. To construct this compactification we first enlarge $\mathbb{H}_F$. We denote $G(\mathbb{Q})^+ := G(\mathbb{Q}) \cap G_\infty^+$ and let $G^{\mathrm{ad}}(\mathbb{Q})^+$ be the image of $G(\mathbb{Q})^+$ in $G^{\mathrm{ad}}(\mathbb{Q})$. In [6] it is constructed a space $\overline{\mathbb{H}}_F$ containing $\mathbb{H}_F$, and it is proved that it is a manifold with corners with smooth boundary. There is a continuous action of $G^{\mathrm{ad}}(\mathbb{Q})^+$ on $\overline{\mathbb{H}}_F$ extending the action over $\mathbb{H}_F$, moreover if Γ is a torsion free arithmetic subgroup of $G^{\mathrm{ad}}(\mathbb{Q})^+$ then $\Gamma \setminus \overline{\mathbb{H}}_F$ is a compact surface with fundamental group Γ. More explicitly we have:

$$\overline{\mathbb{H}}_F := \mathbb{H}_F \sqcup \bigsqcup_P e(P),$$

where the second union is over the set of Borel groups of G^{ad}, and each $e(P)$ is a contractible space. Moreover the boundary $\bigsqcup_P e(P)$ is stable under the action of $G^{\mathrm{ad}}(\mathbb{Q})^+$, in fact we have: $\gamma e(P) = e(\gamma P \gamma^{-1})$ for each $\gamma \in G^{\mathrm{ad}}(\mathbb{Q})^+$ and Borel group P.

Using these notations we define the *Borel–Serre compactification* of Y_K by:

$$X_K := \bigsqcup_{\mathbf{x} \in \mathcal{C}_K^+} \overline{\Gamma}_{\mathbf{x}} \setminus \overline{\mathbb{H}}_F.$$

2.3. Cohomology and Hecke operators

2.3.1. Cohomology

Let M be a module with a right action of K, and suppose that $K \cap Z(\mathbb{Q})$ acts trivially. We denote by $\mathcal{L}(M)$ the sheaf over Y_K given by the local system $G(\mathbb{Q}) \setminus (G(\mathbb{A}) \times M)/KK_\infty^+ Z_\infty \to Y_K$ which is defined by $\gamma(g,m)kk_\infty = (\gamma g k k_\infty, m \cdot k)$. For some specific modules M we will be interested in the cohomology groups: $H^i(Y_K, \mathcal{L}(M))$ and $H^i_c(Y_K, \mathcal{L}(M))$. Suppose that the action of K on M factorizes through the image of K into $G(\mathbb{Q}_p)$, then the groups $\overline{\Gamma}_{\mathbf{x}}$ act on M and so for $? \in \{\emptyset, c\}$ we have the following decomposition:

$$H^i_?(Y_K, \mathcal{L}(M)) = \oplus_{\mathbf{x}\in\mathcal{C}_K^+} H^i_?(\overline{\Gamma}_{\mathbf{x}} \setminus \mathbb{H}_F, \mathcal{L}(M)) = \oplus_{\mathbf{x}\in\mathcal{C}_K^+} H^i_?(\overline{\Gamma}_{\mathbf{x}}, M),$$

where the term in the middle $\mathcal{L}(M)$ is the sheaf over $\overline{\Gamma}_{\mathbf{x}} \setminus \mathbb{H}_F$ given by the local system defined by $\gamma(z,m) = (\gamma z, m\gamma^{-1})$ for $\gamma \in \overline{\Gamma}_{\mathbf{x}}$, $z \in \mathbb{H}_F$ and $m \in M$. Let Γ be a torsion free arithmetic subgroup of $G^{\mathrm{ad}}(\mathbb{Q})^+$ then we have the following exact sequence:

$$\cdots \longrightarrow H^i_c(\Gamma \setminus \mathbb{H}_F, \mathcal{L}(M)) \longrightarrow H^i(\Gamma \setminus \mathbb{H}_F, \mathcal{L}(M)) \longrightarrow H^i(\Gamma \setminus \partial\overline{\mathbb{H}}_F, \mathcal{L}(M)) \longrightarrow \cdots$$

here the sheaf $\mathcal{L}(M)$ on $\Gamma \setminus \partial\overline{\mathbb{H}}_F$ is defined as before. The boundary of $\Gamma\setminus\overline{\mathbb{H}}_F$ is given by $\sqcup_{P\in\mathcal{B}_\Gamma} \Gamma_P \setminus e(P)$ where $\mathcal{B}_\Gamma$ is a fixed set of representatives of the classes of the Borel groups under the action by conjugation of Γ and $\Gamma_P = \Gamma \cap P$. Then we obtain:

$$\begin{aligned} H^\bullet(\partial(\Gamma \setminus \overline{\mathbb{H}}_F), \mathcal{L}_\Gamma(M)) &\simeq \bigoplus_{P\in\mathcal{B}_\Gamma} H^\bullet(\Gamma_P \setminus e(P), \mathcal{L}_{\Gamma_P}(M)) \\ &\simeq \bigoplus_{P\in\mathcal{B}_\Gamma} H^\bullet(\Gamma_P, M). \end{aligned} \tag{2.1}$$

2.3.2. Hecke operators

Let $\Lambda \subset G^{\mathrm{ad}}(\mathbb{Q})^+$ be a semigroup acting on M, Γ, $\Gamma' \subset \Lambda$ be as before, and $\lambda \in \Lambda$ such that $\Gamma' \cap \lambda\Gamma\lambda^{-1}$ is of finite index in Γ'. For $? \in \{\emptyset, c\}$ we define:

$$[\Gamma\gamma\Gamma'] : H^i_?(\Gamma \setminus \mathbb{H}_F, M) \longrightarrow H^i_?(\Gamma' \setminus \mathbb{H}_F, M),$$

by $[\Gamma\lambda\Gamma'] = \mathrm{Cor}^{\Gamma'}_{\Gamma'\cap\lambda\Gamma\lambda^{-1}} \circ [\lambda] \circ \mathrm{res}^{\Gamma}_{\Gamma\cap\lambda^{-1}\Gamma'\lambda}$ here $\mathrm{Cor}^{\Gamma'}_{\Gamma'\cap\lambda\Gamma\lambda^{-1}}$ and $\mathrm{res}^{\Gamma}_{\Gamma\cap\lambda^{-1}\Gamma'\lambda}$ are the classical co-restriction and restriction map on the cohomology and $[\lambda] : H^i_?(\Gamma\cap\lambda^{-1}\Gamma'\lambda\setminus\mathbb{H}_F, M) \to H^i_?(\Gamma'\cap\lambda\Gamma\lambda^{-1}\setminus\mathbb{H}_F, M)$ is given by the map $M \to M$, $m \mapsto m\lambda$. In the same way, we define Hecke operators in the adelic point of view. Let $R \subset G(\mathbb{A}_f)$ be a semigroup acting on M. Let $K \subset G(\mathbb{A}_f)$ be an open compact subgroup contained in R and satisfying 2.1. For each $x \in R$ we put $[KxK] = \mathrm{Cor}_{K\cap xKx^{-1},K} \circ [x] \circ \mathrm{res}_{K,K\cap x^{-1}Kx}$:

$$[KxK] : H^i_?(Y_K, \mathcal{L}(M)) \longrightarrow H^i_?(Y_K, \mathcal{L}(M))$$

where $\mathrm{Cor}_{K\cap xKx^{-1},K}$ and $\mathrm{res}_{K,K\cap x^{-1}Kx}$ are as before, moreover, the morphism $[x]: H^i_?(Y_{K\cap x^{-1}Kx},\mathcal{L}(M)) \to H^i_?(Y_{K\cap xKx^{-1}},\mathcal{L}(M))$ is given as follow: let $\chi : Y_{K\cap xKx^{-1}} \to Y_{K\cap x^{-1}Kx}$ be given by $g\in G(\mathbb{A}) \to gx \in G(\mathbb{A})$, then $[x]$ is the morphism obtained using the χ-cohomomorphism $\mathcal{L}_{K\cap x^{-1}Kx}(M) \rightsquigarrow \mathcal{L}_{K\cap xKx^{-1}}(M)$ (in the notations of [7]) which is defined by $(g,m)\to(gx^{-1},xm)$.

Denote by Λ_2 the image of R in $G(\mathbb{Q}_p)$, then let $\Lambda_1 \subset G(\mathbb{Q})^+$ be the inverse image of Λ_2 under the map $G(\mathbb{Q})^+ \to G(\mathbb{Q}_p)$ and finally let $\Lambda \subset G^{\mathrm{ad}}(\mathbb{Q})^+$ be the image of Λ_1 in $G^{\mathrm{ad}}(\mathbb{Q})^+$. Then Λ acts on M and $\overline{\Gamma}_{\mathbf{y}} \subset \Lambda$ for all $\mathbf{y}\in\mathcal{C}_K^+$. We suppose that $x\in R$ satisfy $\det(K)=\det(K\cap x^{-1}Kx)$ and let $\sigma : \mathcal{C}_K^+ \to \mathcal{C}_K^+$ be the bijection such that for each $\mathbf{y}\in\mathcal{C}_K^+$ we can write $g_{\mathbf{y}}x = \lambda_{\mathbf{y}} g_{\sigma(\mathbf{y})}kc$ where $\lambda_{\mathbf{y}}\in G(\mathbb{Q})$, $k\in K$ and $c\in G_\infty^+$. Then using the above notations we have:

$$[KxK] = \bigoplus_{\mathbf{y}\in\mathcal{C}_K^+}[\overline{\Gamma}_{\sigma(\mathbf{y})}\lambda_{\mathbf{y}}\overline{\Gamma}_{\mathbf{y}}]. \tag{2.2}$$

Notation 2.3. — We use the usual notations about Hecke operators. If $\left(\begin{smallmatrix}1&0\\0&\varpi_{\mathfrak{p}}\end{smallmatrix}\right)\in R$ we denote $U_{\mathfrak{p}}$ the operator $\left[K\left(\begin{smallmatrix}1&0\\0&\varpi_{\mathfrak{p}}\end{smallmatrix}\right)K\right]$ and if $\left(\begin{smallmatrix}1&0\\0&p\end{smallmatrix}\right)\in R$ the we write $U_p=\left[K\left(\begin{smallmatrix}1&0\\0&p\end{smallmatrix}\right)K\right]$. We have $U_p=\prod_{\mathfrak{p}|p}U_{\mathfrak{p}}^{e_{\mathfrak{p}}}$.

2.4. Algebraic representations of G

Through all this work we fix $(\mathbf{k},r)\in\mathbb{Z}^{\Sigma_F}\times\mathbb{Z}$ such that $k_\sigma\geqslant 2$, $k_\sigma\equiv r \bmod 2$ and $\mid r\mid\leqslant k_\sigma-2$ for all $\sigma\in\Sigma_F$. Let L be a finite extension of $\mathbb{Q}_p$ containing a normal closure of F. Attached to the data $(\mathbf{k},r)$ we get a dominant character, λ, for (G_L,B_L,T_L), corresponding to $(\frac{k_\sigma-2+r}{2},\frac{-k_\sigma+2+r}{2})_{\sigma\in\Sigma_F}\in(\mathbb{Z}\times\mathbb{Z})^{\Sigma_F}$ i.e. if A is a L-algebra then for $t=((\begin{smallmatrix}a_\sigma&0\\0&b_\sigma\end{smallmatrix}))_{\sigma\in\Sigma_F}\in T(A)$ we have:

$$\lambda(t)=\prod_{\sigma\in\Sigma_F}a_\sigma^{\frac{k_\sigma-2+r}{2}}b_\sigma^{\frac{-k_\sigma+2+r}{2}}.$$

Let $\mathbb{V}_\lambda$ be the irreducible algebraic representation of highest weight λ of G_L, then $\mathbb{V}_\lambda$ is the algebraic induction of λ from the Borel subgroup of the lower triangular matrices to G_L. We have an explicit description of this representation. Let A be a L-algebra then we have:

$$\mathbb{V}_\lambda(A)\cong\bigotimes_{\sigma\in\Sigma_F}\mathrm{Sym}^{k_\sigma-2}(A)\otimes\det{}^{\frac{r-k_\sigma+2}{2}},$$

here the action is given by: $(g\cdot P)(X,Y)=(\det(g))^{\frac{r\mathbf{t}-\mathbf{k}+2\mathbf{t}}{2}}P(aX-cY,-bX+dY)$, for $g=(\begin{smallmatrix}a&b\\c&d\end{smallmatrix})\in G(A)\simeq\mathrm{Gl}_2(A)^{\Sigma_F}$ and $P(X,Y)$ is a polynomial in the variables $X = (X_\sigma)_{\sigma\in\Sigma_F}$ and $Y = (Y_\sigma)_{\sigma\in\Sigma_F}$ which is homogeneous of degree $k_\sigma - 2$ in the variables X_σ and Y_σ. We denote $L(\mathbf{k}, r; L) = \bigotimes_{\sigma\in\Sigma_F} \mathrm{Sym}^{k_\sigma-2}(L) \otimes \det^{\frac{-r-k_\sigma+2}{2}}$, regarded as a left $G(L)$-module. Let $\mathrm{crit} : L(\mathbf{k}, r; L) \to L$ be the morphism such that $P(X, Y) \in L(\mathbf{k}, r; L)$ is sent to the coefficient in front of $X^{\frac{\mathbf{k}-2\mathbf{t}-r\mathbf{t}}{2}} Y^{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}}$. Consider the morphism $\mathrm{ec} : \mathbb{V}_\lambda^\vee(L) \to L$ given by $\varphi \mapsto \binom{\mathbf{k}-2\mathbf{t}}{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}}\varphi(f_\star)$, where $f_\star \in \mathbb{V}_\lambda(L)$ is uniquely determined by the condition $f_\star\left(\begin{smallmatrix}1 & z\\ 0 & 1\end{smallmatrix}\right) = z^{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}}$ for all $z \in F \otimes_{\mathbb{Q}} L$. Then we have the following commutative diagram:

$$\begin{tikzcd} \mathbb{V}_\lambda^\vee(L) \arrow[r, "\mathrm{ec}"] \arrow[d, "\sim"] & L \\ L(\mathbf{k}, r; L) \arrow[ur, "\mathrm{crit}"'] & \end{tikzcd} \tag{2.3}$$

3. Slope decomposition for the compactly supported cohomology

In this section we prove the following theorem. Let $R \subset G(\mathbb{A}_f)$ be a semigroup and $K \subset G(\mathbb{A}_f)$ be an open compact subgroup contained in R satisfying hypothesis 2.1. Let M be a compact Frechet space over L equipped with a continuous left action of R, where L is a finite extension of $\mathbb{Q}_p$. Suppose that the action of R on M factorizes through the image of R in $G(\mathbb{Q}_p)$ and moreover $K \cap Z(\mathbb{Q})$ acts trivially.

Theorem 3.1. — *Let $x \in R$. Suppose that the action of x on M gives a completely continuous operator on M. Then for each $h \in \mathbb{Q}$ and $i \in \mathbb{N}$ there is a $\leqslant h$-decomposition of $H_c^i(Y_K, \mathcal{L}(M))$ with respect to $[KxK]$.*

This kind of result was proved in [22] to $H^i(Y_K, \mathcal{L}(M))$. In this work we adapt the strategy used in [22], to the cohomology of the boundary of the Borel–Serre compactification of Y_K. Finally using the mapping cone we can prove the theorem.

3.1. Complexes

3.1.1.

Let Γ be a torsion free arithmetic subgroup of $G^{\mathrm{ad}}(\mathbb{Q})^+$. The compact variety $\Gamma\backslash\overline{\mathbb{H}}_F$ is a smooth C^∞-variety with corners, then by [16] we can find a finite triangulation of $\Gamma\backslash\overline{\mathbb{H}}_F$ inducing a triangulation on its boundary. We fix one of those triangulations. Using the natural projection $\overline{\mathbb{H}}_F \to \Gamma \setminus \overline{\mathbb{H}}_F$ we obtain a triangulation on $\overline{\mathbb{H}}_F$. For each $i \in \{0, \dots, 2d\}$ we denote by $\triangle_i$ the set of simplexes of degree i of this last triangulation. The group Γ acts on $\triangle_i$, and the quotient by this action is a finite set, in addition each orbit is in bijection with Γ. Let $C_i(\Gamma) := \mathbb{Z}[\triangle_i]$ be the free $\mathbb{Z}$-module generated by $\triangle_i$. Then $C_i(\Gamma)$ is a free $\mathbb{Z}[\Gamma]$-module of finite rank and by considering the standard boundary operators we obtain the following exact sequence of $\mathbb{Z}[\Gamma]$-modules:

$$0 \longrightarrow C_{2d}(\Gamma) \longrightarrow \cdots \longrightarrow C_1(\Gamma) \longrightarrow C_0(\Gamma) \longrightarrow \mathbb{Z} \longrightarrow 0.$$

When M is a left $\mathbb{Z}[\Gamma]$-module we define $C^\bullet(\Gamma, M) := \mathrm{Hom}_\Gamma(C_\bullet(\Gamma), M)$, then:

- The cohomology of $C^\bullet(\Gamma, M)$ compute the cohomology of Γ i.e. the groups $H^\bullet(\Gamma, M)$;
- $C^i(\Gamma, M)$ is isomorphic to M^{r_i}, here r_i is the number of orbits of the action of Γ on $\triangle_i$.

3.1.2. Complexes from the boundary

From the triangulation of $\overline{\mathbb{H}}_F$ fixed in 3.1.1 we obtain a triangulation of $\partial\overline{\mathbb{H}}_F$. If we call $\triangle_i^\partial$ the set of i-simplexes of this triangulation then in the same way that in 3.1.1 we consider the $\mathbb{Z}[\Gamma]$-modules:

$$C_i^\partial(\Gamma) := \mathbb{Z}[\triangle_i^\partial].$$

Let $\mathcal{B}$ be the set of Borel groups of G^{ad} and we denote by $\mathbb{Z}[\mathcal{B}]$ the free $\mathbb{Z}$-module over $\mathcal{B}$. The group Γ acts on $\mathcal{B}$ by conjugation, then $\mathbb{Z}[\mathcal{B}]$ is in fact a $\mathbb{Z}[\Gamma]$-module.

Proposition 3.2. — *The $\mathbb{Z}[\Gamma]$-module $C_i^\partial(\Gamma)$ is free of finite rank. Moreover $C_\bullet^\partial(\Gamma)$ is a resolution of $\mathbb{Z}[\mathcal{B}]$, i.e. we have the following exact sequence of $\mathbb{Z}[\Gamma]$-modules:*

$$0 \longrightarrow C_{2d-1}^\partial(\Gamma) \longrightarrow \cdots \longrightarrow C_1^\partial(\Gamma) \longrightarrow C_0^\partial(\Gamma) \longrightarrow \mathbb{Z}[\mathcal{B}] \longrightarrow 0. \tag{3.1}$$

Proof. — The first affirmation is a consequence of the fact that the action of Γ on $\partial\overline{\mathbb{H}}_F$ is free and the triangulation of $\Gamma\backslash\overline{\mathbb{H}}_F$ fixed in 3.1.1 is finite. By construction, the complex computes the homology of $\partial\overline{\mathbb{H}}_F$. Moreover we have a decomposition $\partial\overline{\mathbb{H}}_F = \bigsqcup_{P\in\mathcal{B}} e(P)$ (see 2.2.1) where each $e(P)$ is a contractible topological space, then $H_i(\partial\overline{\mathbb{H}}_F) = 0$ if $i > 0$ and $H_0(\partial\overline{\mathbb{H}}_F) = \mathbb{Z}[\mathcal{B}]$. So we deduce the exact sequence (3.1). $\square$

If M is a module with an action of Γ then for each $i \in \mathbb{N}$ we define

$$C^i_\partial(\Gamma, M) := \mathrm{Hom}_\Gamma(C^\partial_i(\Gamma), M).$$

From (3.1) we deduce that $C^\bullet_\partial(\Gamma, M)$ is a complex. In Proposition 3.4 we give other description of the complex $C^\bullet_\partial(\Gamma, M)$. Let P be a Borel of G^{ad}, let $e(P)$ be the contractible space attached to P (see 2.2.1) and denote $\Gamma_P := \Gamma \cap P(\mathbb{Q})$. The triangulation of $\overline{\mathbb{H}}_F$ fixed in Section 3.1.1 induces a triangulation of $e(P)$, then for $i \in \{0, \dots, 2d-1\}$ we denote by $\triangle^P_i$ the set of simplexes of dimension i in this triangulation. In fact, we have $\triangle^P_i = \{s \in \triangle_i \mid s \subset e(P)\}$. We denote $C_i(\Gamma_P)$ the free $\mathbb{Z}$-module over $\triangle^P_i$, then we have:

Lemma 3.3. — *For each i the module $C_i(\Gamma_P)$ is a free $\mathbb{Z}[\Gamma_P]$-module of finite rank. Moreover we have the following exact sequence of $\mathbb{Z}[\Gamma_P]$-modules:*

$$0 \longrightarrow C_{2d-1}(\Gamma_P) \longrightarrow \cdots \longrightarrow C_1(\Gamma_P) \longrightarrow C_0(\Gamma_P) \longrightarrow \mathbb{Z} \longrightarrow 0. \tag{3.2}$$

Proof. — This lemma is proved in the same way as Proposition 3.2. □

If M is a left $\mathbb{Z}[\Gamma_P]$-module we define $C^\bullet(\Gamma_P, M) := \mathrm{Hom}_{\Gamma_P}(C_\bullet(\Gamma_P), M)$. Let $\mathcal{B}_\Gamma$ be a fixed set of representatives of the classes of the Borel groups under the action by conjugation of Γ.

Proposition 3.4. — *We have:*

(1) *For each $\mathbb{Z}[\Gamma]$-module M we have an isomorphism:*

$$C^\bullet_\partial(\Gamma, M) \xrightarrow{\sim} \oplus_{P \in \mathcal{B}_\Gamma} C^\bullet(\Gamma_P, M),$$

this isomorphism is functorial in M.

(2) *Each $C^i_\partial(\Gamma, M)$ is isomorphic to finitely many copies of M. Moreover the cohomology groups $H^\bullet(\Gamma\backslash\partial\overline{\mathbb{H}}_F, \mathcal{L}(M))$ are calculated by taking the cohomology of the complex $C^\bullet_\partial(\Gamma, M)$.*

Proof. — We have the following decomposition of $\mathbb{Z}[\Gamma]$-modules:

$$C^\partial_\bullet(\Gamma) = \bigoplus_{P \in \mathcal{B}_\Gamma} \bigoplus_{Q \sim P} C_\bullet(\Gamma_Q),$$

then is enough to define for each $P \in \mathcal{B}_\Gamma$ an isomorphism:

$$\mathrm{Hom}_\Gamma(\textstyle\bigoplus_{Q\sim P} C_\bullet(\Gamma_Q), M) \xrightarrow{\sim} C^\bullet(\Gamma_P, M).$$

Fix $P \in \mathcal{B}_\Gamma$. We define $\mathrm{Hom}_\Gamma(\bigoplus_{Q\sim P} C_\bullet(\Gamma_Q), M) \to C^\bullet(\Gamma_P, M)$ by $\varphi \to \varphi|_{C_\bullet(\Gamma_P)}$. We will verify that this map is an isomorphism. Let φ such that $\varphi|_{C_\bullet(\Gamma_P)} = 0$. Let $Q \sim P$ and $s \in C_\bullet(\Gamma_Q)$. There exist $\gamma \in \Gamma$ such that $\gamma s \in C_\bullet(\Gamma_P)$, then $\varphi(s) = \gamma^{-1}\varphi(\gamma s) = 0$. So $\varphi = 0$ and then the morphism is injective. To prove that it is surjective, let $\varphi \in C^\bullet(\Gamma_P, M)$ and we define $\overline{\varphi} : \bigoplus_{Q\sim P} C_\bullet(\Gamma_Q) \to M$ as follows: let $s \in C_\bullet(\Gamma_Q)$ where $Q \sim P$ and we choice $\gamma \in \Gamma$ such that $\gamma s \in C_\bullet(\Gamma_P)$, then we define $\overline{\varphi}(s) := \gamma^{-1}\varphi(\gamma s)$. Is not difficult to prove that $\overline{\varphi}$ is well defined and is Γ-equivariant.

The first affirmation of part (2) is deduced from the fact that $C^\partial_\bullet(\Gamma)$ is a free $\mathbb{Z}[\Gamma]$-module of finite rank. Finally from part (1) and decomposition (2.1) we deduce that the cohomology of $C^\bullet_\partial(\Gamma, M)$ is $H^\bullet(\Gamma \setminus \partial\overline{\mathbb{H}}_F, \mathcal{L}(M))$. □

3.1.3. Compact supports

We recall de notion of *mapping cone of a morphism of complexes*. Let $\mathcal{A}$ be an abelian category. If $\pi = \pi^\bullet : C^\bullet \to D^\bullet$ is a morphism of complexes of elements of $\mathcal{A}$, we obtain a new complex of elements in $\mathcal{A}$ denoted by $\mathrm{Cone}(\pi)^\bullet$ and defined as follows: for each $i \in \mathbb{Z}$ we have $\mathrm{Cone}(\pi)^i := C^i \oplus D^{i-1}$ and the differential is defined by:

$$\begin{aligned} d : \mathrm{Cone}(\pi)^i &\longrightarrow \mathrm{Cone}(\pi)^{i+1}\ , \\ (c, d) &\longmapsto (-d_C(c), -\pi^i(c) + d_D(d)). \end{aligned}$$

If $\Gamma \subset G^{\mathrm{ad}}(\mathbb{Q})$ and M a Γ-module we denote $\pi^\bullet : C^\bullet(\Gamma, M) \to C^\bullet_\partial(\Gamma, M)$ the morphism of complexes obtained from the inclusion $C^\partial_\bullet(\Gamma) \subset C_\bullet(\Gamma)$. We define:

$$C^\bullet_c(\Gamma, M) := \mathrm{Cone}(\pi)^\bullet.$$

Proposition 3.5. — *Each $C^i_c(\Gamma, M)$ is isomorphic to finitely many copies of M. Moreover the cohomology groups $H^\bullet_c(\Gamma \setminus \mathbb{H}_F, \mathcal{L}(M))$ are calculated by taking the cohomology of the complex $C^\bullet_c(\Gamma, M)$.*

Proof. — The first assertion is a direct consequence of 3.1.1 and Proposition 3.4. We have two long exact sequences:

$$\begin{aligned} \cdots \longrightarrow H^i_c(\Gamma \setminus \mathbb{H}_F, \mathcal{L}(M)) \longrightarrow H^i(\Gamma \setminus \mathbb{H}_F, \mathcal{L}(M)) \\ \longrightarrow H^i(\Gamma \setminus \partial\mathbb{H}_F, \mathcal{L}(M)) \longrightarrow \cdots \end{aligned}$$

and

$$\cdots \longrightarrow H^i(C^\bullet_c(\Gamma, M)) \longrightarrow H^i(C^\bullet(\Gamma, M)) \longrightarrow H^i(C^\bullet_\partial(\Gamma, M)) \longrightarrow \cdots$$

Moreover, for each i we have $H^i(\Gamma\setminus\mathbb{H}_F, \mathcal{L}(M)) \simeq H^i(C^\bullet(\Gamma, M))$ and $H^i(\Gamma\setminus \partial\mathbb{H}_F, \mathcal{L}(M)) \simeq H^i(C^\bullet_\partial(\Gamma, M))$. Then, using the five lemma we obtain the proposition. □

3.2. Hecke operators on complexes

In this subsection we define operators on $C_c^\bullet(\Gamma, M)$ which induces the standard Hecke operators on the cohomology.

3.2.1. Compatible pairs

Let Γ and Γ' be torsion free arithmetic subgroups of $G^{\mathrm{ad}}(\mathbb{Q})^+$ and $\phi : \Gamma \to \Gamma'$ a group homomorphism. We take a left Γ-module N and a left Γ'-module M. A pair (ϕ, α) is called *compatible* if $\alpha : M \to N$ is a morphism of $\mathbb{Z}[\Gamma]$-modules when we consider M as a $\mathbb{Z}[\Gamma]$-module via ϕ. In [22] the author associates a morphism $\alpha^\bullet : C^\bullet(\Gamma', M) \to C^\bullet(\Gamma, N)$ to each such pair. We obtain an analogous morphism in the boundary.

By Proposition 3.2, $C_\bullet^\partial(\Gamma)$ is a projective resolution of $\mathbb{Z}[\mathcal{B}]$ by $\mathbb{Z}[\Gamma]$-modules. Via ϕ we can consider $C_\bullet^\partial(\Gamma')$ as other resolution of $\mathbb{Z}[\mathcal{B}]$ by $\mathbb{Z}[\Gamma]$-modules, then there is a map $\phi_\bullet : C_\bullet^\partial(\Gamma) \to C_\bullet^\partial(\Gamma')$ compatible with ϕ, and it is unique up to homotopy. Then we obtain a map $\alpha^\bullet : C_\partial^\bullet(\Gamma', M) \to C_\partial^\bullet(\Gamma, N)$ given by $\varphi \to \alpha \circ \varphi \circ \phi_\bullet$. This map is uniquely defined up to homotopy. The relation between these constructions is given by:

$$\begin{tikzcd} C_\bullet(\Gamma) \arrow[r] & C_\bullet(\Gamma') \\ C_\bullet^\partial(\Gamma) \arrow[r] \arrow[u] & C_\bullet^\partial(\Gamma') \arrow[u] \end{tikzcd} \qquad \begin{tikzcd} C^\bullet(\Gamma, N) \arrow[d] & C^\bullet(\Gamma', M) \arrow[l] \arrow[d] \\ C_\partial^\bullet(\Gamma, N) & C_\partial^\bullet(\Gamma', M) \arrow[l] \end{tikzcd}$$

the first diagram is in homotopy category on $\mathbb{Z}[\Gamma]$ and the second one in the homotopy category on $\mathbb{Z}$.

- If we take $\Gamma \subset \Gamma'$, $M = N$, ϕ be the inclusion and α the identity, then we obtain the restriction map: $\mathrm{res}_\Gamma^{\Gamma'} : C_\partial^\bullet(\Gamma', M) \to C_\partial^\bullet(\Gamma, M)$.
- Suppose $\Lambda \subset G^{\mathrm{ad}}(\mathbb{Q})^+$ is a semigroup, $\Gamma \subset \Lambda$ and $\lambda \in \Lambda$ such that $\lambda\Gamma\lambda^{-1} \subset \Lambda$. Moreover suppose that M is a left Λ-module. We take $\phi : \lambda\Gamma\lambda^{-1} \to \Gamma$ given by $\gamma \mapsto \lambda^{-1}\gamma\lambda$ and $M \to M$ defined by $m \mapsto \lambda m$. Then obtain a morphism denoted by $[\lambda] : C_\partial^\bullet(\Gamma, M) \to C_\partial^\bullet(\lambda\Gamma\lambda^{-1}, M)$.

3.2.2. Corestriction map

Suppose $\Gamma \subset \Gamma'$ is of finite index. The complex $C_\bullet^\partial(\Gamma')$ is a projective resolution of $\mathbb{Z}[\mathcal{B}]$ by $\mathbb{Z}[\Gamma]$-modules. Then there exists a map $\tau : C_\bullet^\partial(\Gamma') \to C_\bullet^\partial(\Gamma)$ of Γ-modules, unique up to homotopy. If M is a left Γ'-module then we obtain a map $\mathrm{Cor}_{\Gamma}^{\Gamma'} : C_{\partial}^{\bullet}(\Gamma, M) \to C_{\partial}^{\bullet}(\Gamma', M)$ called the *corestriction map* and defined as follows: fix a decomposition $\Gamma' = \sqcup_q \gamma_q \Gamma$ then for any $\varphi \in C_{\partial}^{\bullet}(\Gamma, M)$ we put

$$\mathrm{Cor}_{\Gamma}^{\Gamma'}(\varphi)(s) = \sum_q \gamma_q \varphi(\tau(\gamma_q^{-1} s)).$$

This map is uniquely defined up to homotopy. In [22] is defined a morphism $\mathrm{Cor}_{\Gamma}^{\Gamma'} : C^{\bullet}(\Gamma, M) \to C^{\bullet}(\Gamma', M)$, and we have commutative diagrams as in 3.2.1 relating these two corestriction morphisms.

3.2.3. Hecke operators

Let $\Lambda \subset G^{\mathrm{ad}}(\mathbb{Q})^{+}$ be a semigroup, $\Gamma, \Gamma' \subset \Lambda$ be free torsion arithmetic groups. Let $\lambda \in \Lambda$ be such that $\Gamma' \cap \lambda \Gamma \lambda^{-1}$ is of finite index in Γ'. We define:

$$[\Gamma \lambda \Gamma'] : C_{\partial}^{\bullet}(\Gamma, M) \to C_{\partial}^{\bullet}(\Gamma', M)$$

$$\text{by} \quad [\Gamma \lambda \Gamma'] = \mathrm{Cor}_{\Gamma' \cap \lambda \Gamma \lambda^{-1}}^{\Gamma'} \circ [\lambda] \circ \mathrm{res}_{\Gamma \cap \lambda^{-1} \Gamma' \lambda}^{\Gamma}.$$

In [22] is defined the Hecke operator $[\Gamma \lambda \Gamma'] : C^{\bullet}(\Gamma, M) \to C^{\bullet}(\Gamma', M)$ and we have the following commutative diagram:

$$\begin{tikzcd} C^{\bullet}(\Gamma, M) \arrow[r, "{[\Gamma\lambda\Gamma']}"] \arrow[d] & C^{\bullet}(\Gamma', M) \arrow[d] \\ C_{\partial}^{\bullet}(\Gamma, M) \arrow[r, "{[\Gamma\lambda\Gamma']}"] & C_{\partial}^{\bullet}(\Gamma', M) \end{tikzcd}$$

This diagram live in the homotopy category over $\mathbb{Z}$, in this last category the mapping is well defined and we obtain a morphism:

$$[\Gamma \lambda \Gamma'] : C_{c}^{\bullet}(\Gamma, M) \to C_{c}^{\bullet}(\Gamma', M). \tag{3.3}$$

Remark 3.6. — These morphisms give the usual morphisms on the cohomology.

3.3. Conclusions

Consider the notation on the beginning of this section. On the cohomology we have the decomposition (2.2), then from Subsections 3.1 and 3.2 we obtain immediately the following proposition:

Proposition 3.7. — *Suppose that K satisfies hypothesis 2.1, the action of R on M factorizes through the projection $R \to G(\mathbb{Q}_p)$ and $K \cap Z(\mathbb{Q})$ acts trivially on M. Then, there exists a bounded complex $R\Gamma_c^\bullet(K, M)$ such that:*

(1) *The cohomology of $R\Gamma_c^\bullet(K, M)$ is $H_c^\bullet(Y_K, \mathcal{L}(M))$.*
(2) *Each $R\Gamma_c^i(K, M)$ is isomorphic to finitely many copies of M.*
(3) *We can define operators over $R\Gamma_c^\bullet(K, M)$ giving the classical Hecke operators on $H_c^i(Y_K, \mathcal{L}(M))$.*

Proof of Theorem 3.1. — From Proposition 3.7 and [22, §2.3.13], we deduce the existence of the $\leqslant h$-decomposition of $R\Gamma_c^\bullet(K, M)$ with respect to $[KxK]$ for each $i \in \mathbb{Z}$. Finally we deduce the theorem from the discussions in [22, §2.3.10] and [22, §2.3.12]. □

4. Distributions

In this section we recall the definition of the spaces of distributions that we will use to define the p-adic overconvergent coefficients on Hilbert modular varieties. Moreover we describe the space of distributions on some Galois groups.

4.1. Generalities

4.1.1. Definitions

Let $X \subset \mathbb{Q}_p^r$ be an open compact subset. Let $\mathcal{A}(X, L)$ be the vector space over L of the locally L-analytic functions $f : X \to L$. Let $\mathcal{A}_n(X, L)$ be the subspace of $\mathcal{A}(X, L)$ such that $f \in \mathcal{A}_n(X, L)$ if and only if f is analytic on the disks of radius p^{-n}. The space $\mathcal{A}_n(X, L)$ is a Banach space when equipped with the norm defined as follows. Let $f \in \mathcal{A}_n(X, L)$ and fix a covering of X by disks of radius p^{-n}. Fix one of these disks and let $\mathbf{a} = (a_1, \dots, a_r) \in X$ be one of its centers, over this disk we can write:

$$f(x_1, \dots, x_r) = \sum_{\mathbf{m} \in \mathbb{N}^r} c_{\mathbf{m}}(\mathbf{a})(x_1 - a_1)^{m_1} \dots (x_r - a_r)^{m_r},$$

Then we define $\|f\|_n = \sup\{p^{-n\sum_i m_i}|c_{\mathbf{m}}(\mathbf{a})|_p \mid \mathbf{m} \in \mathbb{N}^r, \mathbf{a}\}$ where $\mathbf{a}$ run through the set of centers of the fixed covering of X. Because X is compact we have $\mathcal{A}(X, L) = \cup_{n \geqslant 0} \mathcal{A}_n(X, L)$ and then we will consider the inductive limit topology on $\mathcal{A}(X,L)$. Let $\mathcal{D}(X,L)$ be the continuous dual of $\mathcal{A}(X,L)$. Moreover, let $\mathcal{D}_n(X,L)$ be the continuous dual of $\mathcal{A}_n(X,L)$, then $\mathcal{D}_n(X,L)$ is a Banach space and $\mathcal{D}(X,L)$ is the projective limit over n of the $\mathcal{D}_n(X,L)$'s. It is possible to prove that the inclusions $\mathcal{A}_n(X,L) \subset \mathcal{A}_{n+1}(X,L)$ are completely continuous (see [22, Lemma 3.2.2]). From this fact we deduce that $\mathcal{D}(X,L)$ is a compact Fréchet space.

4.1.2. Admissibility

Let M be a vector space over L endowed with a decomposition $M = \cup_{n\in\mathbb{N}} M_n$,such that for each n M_n is a Banach space, $M_n \subset M_{n+1}$ and this last inclusion is completely continuous. We consider M with the inductive limit topology of the M_n's. Let $M^\vee$ (resp. $M_n^\vee$) be the continuous dual of M (resp. M_n). If $\mu \in M^\vee$ we denote $\|\mu\|_n$ the number $\|\mu\mid_{M_n}\|_n$, where $\|\cdot\|_n$ is the norm obtained in $M_n^\vee$.

Definition 4.1. — *Let $h \in \mathbb{Q}$. A $\mu \in M^\vee$ is called h-*admissible *if there exists a constant $C > 0$ such that for each n we have $\|\mu\|_n \leqslant Cp^{nh}$.*

Remark 4.2. — If we take $M = \mathcal{A}(X,L)$ and $M_n = \mathcal{A}_n(X,L)$ then $\mu \in \mathcal{D}(X,L)$ is h-admissible if there exist some $C > 0$ such that for each $n \in \mathbb{N}$ and for all $f \in \mathcal{A}_n(X,L)$ we have $\mid \mu \mid_p \leqslant Cp^{nh}\|f\|_n$.

In particular if we put $X = \mathbb{Z}_p$ then we obtain that if $\mu \in \mathcal{D}(\mathbb{Z}_p, L)$ is h-admissible then there exist $C > 0$ such that for all $a \in \mathbb{Z}_p$, $j \in \mathbb{N}$ and $n \in \mathbb{N}$ we have:

$$|\mu(1_{a+p^n\mathbb{Z}_p}(z-a)^j)|_p \leqslant Cp^{n(h-j)}.$$

Compare with [1] and [23].

4.2. Distributions on $\mathcal{O}_F \otimes \mathbb{Z}_p$.

4.2.1. Spaces

We fix an identification of $\mathcal{O}_F \otimes \mathbb{Z}_p$ with an open compact of $\mathbb{Q}_p^d$. Using the notations of 4.1.1 we denote:

$$\mathcal{A}(L) := \mathcal{A}(\mathcal{O}_F \otimes \mathbb{Z}_p, L), \qquad \mathcal{D}(L) := \mathcal{D}(\mathcal{O}_F \otimes \mathbb{Z}_p, L),$$
$$\mathcal{A}_n(L) := \mathcal{A}_n(\mathcal{O}_F \otimes \mathbb{Z}_p, L), \qquad \mathcal{D}_n(L) := \mathcal{D}_n(\mathcal{O}_F \otimes \mathbb{Z}_p, L)$$

and we have

$$\mathcal{D}(L) \to \mathcal{D}_n(L).$$

4.2.2. The action of a semi-group

We define some groups and semi-groups in $G(\mathbb{Q}_p)$:

- $T^+ = \{(\begin{smallmatrix} a & 0 \\ 0 & b \end{smallmatrix}) \in T(\mathbb{Q}_p) \mid \ ba^{-1} \in \mathcal{O}_F \otimes \mathbb{Z}_p\}$
- $T^{++} = \{(\begin{smallmatrix} a & 0 \\ 0 & b \end{smallmatrix}) \in T(\mathbb{Q}_p) \mid \ ba^{-1} \in p\mathcal{O}_F \otimes \mathbb{Z}_p\}$.
- $I = \{(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in G(\mathbb{Z}_p) \mid c \in p\mathcal{O}_F \otimes \mathbb{Z}_p\}$
- $\Lambda_p := IT^+I = G(\mathbb{Q}_p) \cap \{x(\begin{smallmatrix} a & b \\ pc & d \end{smallmatrix}) \mid a \in (\mathcal{O}_F \otimes \mathbb{Z}_p)^\times, b, c, d \in \mathcal{O}_F \otimes \mathbb{Z}_p,$ $x \in (F \otimes \mathbb{Q}_p)^\times\}$

We define an action of Λ_p on $\mathcal{A}(L)$ and $\mathcal{D}(L)$. For $f \in \mathcal{A}(L)$, $\gamma \in \Lambda_p$ and $z \in \mathcal{O}_F \otimes \mathbb{Z}_p$ we put:

- If $\gamma = (\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in I$ let:
$$(\gamma * f)(z) = \lambda \begin{pmatrix} (a-cz) & 0 \\ 0 & \det(\gamma)(a-cz)^{-1} \end{pmatrix} f\left(\frac{-b+dz}{a-cz}\right).$$
- If $\gamma = (\begin{smallmatrix} a & 0 \\ 0 & d \end{smallmatrix}) \in T^+$ let:
$$(\gamma * f)(z) = \lambda \begin{pmatrix} u(a) & 0 \\ 0 & u(d) \end{pmatrix} f(da^{-1}z),$$
see Subsection 2.1 to the definition of the function u.

This definition gives us a well defined continuous action on $\mathcal{A}(L)$, simply because we made explicit the action defined in [22]. The space $\mathcal{A}_n(L)$ is stable under this action. Then we obtain a continuous action on $\mathcal{D}(L)$ and $\mathcal{D}_n(L)$. These spaces endowed with this action are noted by $\mathcal{A}_\lambda(L)$, $\mathcal{A}_{\lambda,n}(L)$, $\mathcal{D}_\lambda(L)$ and $\mathcal{D}_{\lambda,n}(L)$.

Lemma 4.3. — *Let $\gamma \in T^{++}$ then the morphism defined on $\mathcal{D}_\lambda(L)$ is a compact operator.*

Proof. — See [22, Lemma 3.2.8]. □

Remark 4.4. — Depending of the situation we will use the right action of Λ_p on $\mathcal{D}(L)$ or the left action of Λ_p^{-1}.

4.2.3. $\mathcal{O}_L$-modules

Let $\mathcal{A}(\mathcal{O}_L)$ be the $\mathcal{O}_L$-module of the functions $f \in \mathcal{A}(L)$ with values in $\mathcal{O}_L$. The topology in $\mathcal{A}(L)$ induces a topology on $\mathcal{A}(\mathcal{O}_L)$ and we denote by $\mathcal{D}(\mathcal{O}_L)$ its continuous dual. We have $\mathcal{A}(L) = L\mathcal{A}(\mathcal{O}_L)$ then we can consider $\mathcal{D}(\mathcal{O}_L)$ as a $\mathcal{O}_L$-sub-module of $\mathcal{D}(L)$ in the natural way. It is important to remark that $\mathcal{A}(\mathcal{O}_L)$ is stable under the action of Λ_p and then we obtain a right action of Λ_p on $\mathcal{D}(\mathcal{O}_L)$. As before we write $\mathcal{A}_\lambda(\mathcal{O}_L)$

and $\mathcal{D}_\lambda(\mathcal{O}_L)$ when we consider the action of Λ_p on these modules. We denote $\mathcal{A}_{\lambda,n}(\mathcal{O}_L) := \mathcal{A}_{\lambda,n}(L) \cap \mathcal{A}_\lambda(\mathcal{O}_L)$. This $\mathcal{O}_L$-module is stable under the action of Λ_p. We consider the induced topology on it and we call its continuous dual by $\mathcal{D}_{\lambda,n}(\mathcal{O}_L)$. Moreover, we have the restriction morphism $\mathcal{D}_\lambda(\mathcal{O}_L) \to \mathcal{D}_{\lambda,n}(\mathcal{O}_L)$, it is continuous and compatible with the action of Λ_p.

Remark 4.5. — For each n the space $\mathcal{D}_{\lambda,n}(L)$ is a Banach L-vector space and $\mathcal{D}_{\lambda,n}(\mathcal{O}_L) \otimes L = \mathcal{D}_{\lambda,n}(L)$. But is important to remark that $\mathcal{D}_\lambda(L)$ is just a compact Frechet and $\mathcal{D}_\lambda(\mathcal{O}_L) \otimes L \neq \mathcal{D}_\lambda(L)$.

4.2.4.

Let $\mathbb{V}_\lambda(L) \hookrightarrow \mathcal{A}_\lambda(L)$, $f \mapsto \hat{f}$ be given by $\hat{f}(z) = f\left(\begin{smallmatrix} 1 & z \\ 0 & 1 \end{smallmatrix}\right)$ for each $z \in \mathbb{Z}_p \otimes \mathcal{O}_F$, here we consider $\mathbb{V}_\lambda(L)$ as the algebraic induction of λ. This map is a continuous homomorphism, then we obtain $\pi : \mathcal{D}_\lambda(L) \to \mathbb{V}_\lambda(L)^\vee$. Moreover, it is I-equivariant map but not Λ_p-equivariant, in fact for any $\mu \in \mathcal{D}_\lambda(L)$ we have:

$$\pi\left(\mu * \begin{pmatrix} 1 & 0 \\ 0 & \varpi_\mathfrak{p} \end{pmatrix}\right) = \varpi_\mathfrak{p}^{\sum_{\sigma\in\Sigma_\mathfrak{p}} \frac{k_\sigma - 2 - r}{2}} \pi(\mu) \cdot \begin{pmatrix} 1 & 0 \\ 0 & \varpi_\mathfrak{p} \end{pmatrix},$$

4.2.5. Invariant distributions

Here we define a space of distributions that will be very useful to define evaluations on the overconvergent cohomology. Let $E(1)$ be the sub-group of $\mathcal{O}_F^\times$ of totally positive units. We denote by $\mathcal{A}_\lambda^+(L)$ the space of $f \in \mathcal{A}_\lambda(L)$ such that $f * \left(\begin{smallmatrix} e & 0 \\ 0 & 1 \end{smallmatrix}\right) = f$ for all $e \in E(1)$. In fact $f \in \mathcal{A}_\lambda^+(L)$ if and only if $f(ez) = \lambda\left(\begin{smallmatrix} e & 0 \\ 0 & 1 \end{smallmatrix}\right) f(z)$ for any $e \in E(1)$ and $z \in \mathcal{O}_F \otimes \mathbb{Z}_p$. We can verify that $\mathcal{A}_\lambda^+(L)$ is stable under the action of T^+. Moreover, the space $\mathcal{A}_\lambda^+(L)$ is a Frechet and we have $\mathcal{A}_\lambda^+(L) = \cup_{n\in\mathbb{N}} \mathcal{A}_{\lambda,n}^+(L)$, here $\mathcal{A}_{\lambda,n}^+(L)$ is the Banach space given by $\mathcal{A}_\lambda^+(L) \cap \mathcal{A}_{\lambda,n}(L)$. Let $\mathcal{D}_\lambda^+(L)$ be the continuous dual of $\mathcal{A}_\lambda^+(L)$. Then $\mathcal{D}_\lambda^+(L)$ is a compact Frechet and it is endowed with a continuous right action of T^+.

4.3. Distributions on Galois groups

We describe the space of distributions where we find the p-adic L-functions.

4.3.1. Galois groups

Let $F^{p,\infty}$ be the maximal abelian extension of F, unramified outside p and ∞. We denote

$$\mathrm{Gal}_p = Gal(F^{p,\infty}/F).$$

Let F^1 be the narrow class field of F, by definition we have $\mathrm{Gal}(F^1/F) \simeq \mathrm{Cl}_F^+$. Then we have a natural morphism $\mathrm{Gal}_p \to \mathrm{Cl}_F^+$. The kernel of this morphism described using by class field theory: we consider $E(1)$ inside $(\mathcal{O}_F \otimes \mathbb{Z}_p)^\times$ in the natural way, then we have an exact sequence of topological groups:

$$0 \longrightarrow (\mathcal{O}_F \otimes \mathbb{Z}_p)^\times/\overline{E(1)} \xrightarrow{r} \mathrm{Gal}_p \longrightarrow \mathrm{Cl}_F^+ \longrightarrow 0. \tag{4.1}$$

Then we obtain a natural decomposition $\mathrm{Gal}_p = \coprod_{\mathbf{x}\in\mathrm{Cl}_F^+} \mathrm{Gal}_{p,\mathbf{x}}$. If we choose for each $\mathbf{x} \in \mathrm{Cl}_F^+$ a $\sigma_{\mathbf{x}} \in \mathrm{Gal}_{p,\mathbf{x}}$, then the last exact sequence gives us a homeomorphism:

$$r_{\mathbf{x}} : (\mathcal{O}_F \otimes \mathbb{Z}_p)^\times/\overline{E(1)} \longrightarrow \mathrm{Gal}_{p,\mathbf{x}} \tag{4.2}$$

Remark 4.6. — Using Class field Theory we can identify finite order characters of Gal_p with Hecke characters of F of finite order whose conductor contain only primes of F lying above p. In the rest of this paper we freely use this identification.

4.3.2.

Let $\mathrm{Gal}_p^\circ = (\mathcal{O}_F \otimes \mathbb{Z}_p)^\times/\overline{E(1)}$. The topological groups Gal_p° and Gal_p are p-adic spaces, and we fix isomorphisms of these groups with open compact subsets of $\mathbb{Q}_p^{1+\delta}$, where δ is the Leopoldt defect of F. Then using notations in 4.1.1 we can consider the spaces $\mathcal{A}(\mathrm{Gal}_p, L)$, $\mathcal{D}(\mathrm{Gal}_p, L)$, $\mathcal{A}(\mathrm{Gal}_p^\circ, L)$ and $\mathcal{D}(\mathrm{Gal}_p^\circ, L)$; in addition in $\mathcal{D}(\mathrm{Gal}_p^\circ, L)$ and $\mathcal{D}(\mathrm{Gal}_p, L)$ we have the notion of *admissible distribution* (see 4.1.1).

From the decomposition $\mathrm{Gal}_p = \coprod_{\mathbf{x}\in\mathrm{Cl}_F^+} \mathrm{Gal}_{p,\mathbf{x}}$ and (4.2) we obtain an isomorphism of Frechet spaces $\mathcal{A}(\mathrm{Gal}_p, L) \cong \mathcal{A}(\mathrm{Gal}_p^\circ, L)^{\mathrm{Cl}_F^+}$ given by $f \mapsto (f_{\mathbf{x}})_{\mathbf{x}\in\mathrm{Cl}_F^+}$ where $f_{\mathbf{x}} = f \circ r_{\mathbf{x}}$. Then we obtain an isomorphism of compact Frechet spaces:

$$\mathcal{D}(\mathrm{Gal}_p, L) = \mathcal{D}(\mathrm{Gal}_p^\circ, L)^{\mathrm{Cl}_F^+}. \tag{4.3}$$

Remark 4.7. — We can identify the space $\mathcal{A}(\mathrm{Gal}_p^\circ, L)$ with the space of functions $f \in \mathcal{A}((\mathcal{O}_F \otimes \mathbb{Z}_p)^\times, L)$ such that $f(ez) = f(z)$ for all $z \in (\mathcal{O}_F \otimes \mathbb{Z}_p)^\times$ and $e \in E(1)$. So we obtain that:

$$\mathcal{A}(\mathrm{Gal}_p, L) = \left\{ f : \mathrm{Gal}_p \to L \;\middle|\; \begin{array}{r} f \circ r_{\mathbf{x}} \circ \pi \in \mathcal{A}((\mathcal{O}_F \otimes \mathbb{Z}_p)^\times, L) \\ \forall\, \mathbf{x} \in \mathrm{Cl}_F^+ \end{array} \right\}, \tag{4.4}$$

here $\pi : (\mathcal{O}_F \otimes \mathbb{Z}_p)^\times \to \mathrm{Gal}_p^\circ$ is the natural projection.

4.3.3.

Let $\mathcal{A}(\mathrm{Gal}_p^\circ, L) \to \mathcal{A}_\lambda^+(L)$, $f \mapsto \overline{f}$ be defined by:

$$\overline{f}(z) = \begin{cases} 0 & \text{if } z \notin (\mathcal{O}_F \otimes \mathbb{Z}_p)^\times \\ A\lambda\left(\begin{smallmatrix} z & 0 \\ 0 & 1 \end{smallmatrix}\right) f(z) & \text{if } z \in (\mathcal{O}_F \otimes \mathbb{Z}_p)^\times, \end{cases} \tag{4.5}$$

here $A = \binom{\mathbf{k}-2\mathbf{t}}{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}} \in \mathbb{Z}$. This morphism is in fact continuous, then we obtain:

$$\mathcal{D}_\lambda^+(L) \to \mathcal{D}(\mathrm{Gal}_p^\circ, L), \text{ denoted by } \mu \mapsto \mu^\times.$$

Lemma 4.8. — *We have:*

(1) *There exists $C > 0$ depending only on λ such that if $n \geqslant 1$ and $f \in \mathcal{A}_n(\mathrm{Gal}_p^\circ, L)$ then $\overline{f} \in \mathcal{A}_{\lambda,n}^+(L)$ and moreover we have $\|\overline{f}\|_n \leqslant C\|f\|_n$.*

(2) *Let $h \in \mathbb{Q}$. If $\mu \in \mathcal{D}_\lambda^+(L)$ is a h-admissible distribution then $\mu^\times \in \mathcal{D}(\mathrm{Gal}_p^\circ, L)$ is also h-admissible.*

Proof. — Firstly if $f \in \mathcal{A}_n(\mathrm{Gal}_p^\circ, L)$ then is clear that $\overline{f} \in \mathcal{A}_{\lambda,n}^+(L)$ for $n \geqslant 1$. Now let $g_\lambda : \mathcal{O}_F \otimes \mathbb{Z}_p \to L$ be defined by $g_\lambda(z) = A\lambda\left(\begin{smallmatrix} z & 0 \\ 0 & 1 \end{smallmatrix}\right)$. Then $g_\lambda \in \mathcal{A}_{\lambda,0}^+(L)$. If we denote $C = \|g_\lambda\|_0$ then we have $\|\overline{f}\|_n \leqslant C\|f\|_n$. The second part of the lemma is a direct consequence of the first one. □

5. Comparison theorem

Let L be a finite extension of $\mathbb{Q}_p$ containing the normal closure of F. In the next two sections we will exclude L of the spaces defined above. For example we use $\mathbb{V}_\lambda$, $\mathcal{D}_\lambda$, $\mathcal{D}_\lambda^+ \ldots$ instead of $\mathbb{V}_\lambda(L)$, $\mathcal{D}_\lambda(L)$, $\mathcal{D}_\lambda^+(L) \ldots$

Let K be an open compact subgroup of $G(\mathbb{A}_f)$ whose image in $G(\mathbb{Q}_p)$ is contained in Λ_p. The morphism $\mathcal{D}_\lambda \to \mathbb{V}_\lambda^\vee$, described in 4.2.4, is I-equivariant then we obtain a morphism on the cohomology:

$$\pi : H_c^d(Y_K, \mathcal{L}(\mathcal{D}_\lambda)) \longrightarrow H_c^d(Y_K, \mathcal{L}(\mathbb{V}_\lambda^\vee)).$$

On $H^d_c(Y_K, \mathcal{L}(\mathcal{D}_\lambda))$ we consider the $\leqslant$-slope decomposition with respect to U_p (the existence of such decomposition is given by Theorem 3.1). Over $H^d_c(Y_K, \mathcal{L}(\mathbb{V}^\vee_\lambda))$ we consider $\leqslant$-slope decomposition with respect to $U^0_p = p^{\sum_{\sigma\in\Sigma_F} \frac{k_\sigma-2-r}{2}} U_p$. By 4.2.4 we have $\pi \circ U_p = U^0_p \circ \pi$ then π induces a morphism of the $\leqslant$-slope parts. In fact we have the following result:

Theorem 5.1. — *We denote $k^o = \min\{k_\sigma \mid \sigma \in \Sigma_F\}$. If $h \in \mathbb{Q}$ and $h < k^o - 1$ then we have a canonical isomorphism:*

$$H^d_c(Y_K, \mathcal{L}(\mathcal{D}_\lambda))^{\leqslant h} \xrightarrow{\sim} H^d_c(Y_K, \mathcal{L}(\mathbb{V}^\vee_\lambda))^{\leqslant h}.$$

To proof this theorem we follow [22]. The main difficulty to use the approach used in [22] is the existence of the slope decomposition for the compactly supported cohomology, such existence was proved in Section 3. To finish we use the locally analytic version of the BGG-resolution.

5.1. BGG resolution

Let $V_\lambda = V_\lambda(L)$ be the locally algebraic induction as defined in [22, §3.2.9]. As in [22] we can see V_λ within $\mathcal{A}_\lambda$ and it is invariant under the action of Λ_p, then we obtain a Λ_p-equivariant map $\mathcal{D}_\lambda \to V^\vee_\lambda$. We have a canonical inclusion $\mathbb{V}_\lambda \subset V_\lambda$, moreover we have a right action of Λ_p over V_λ, moreover it is I-equivariant. We obtain an I-equivariant morphism $V^\vee_\lambda \to \mathbb{V}^\vee_\lambda$ and then

$$H^d_c(Y_K, \mathcal{L}(V^\vee_\lambda)) \longrightarrow H^d_c(Y_K, \mathcal{L}(\mathbb{V}^\vee_\lambda)).$$

If we consider U_p on the left side and U^0_p on the right side, then this map is compatible with these Hecke operators, moreover from Theorem 3.1 we deduce that $H^d_c(Y_K, \mathcal{L}(V^\vee_\lambda))$ has slope decomposition with respect to U_p. Then in the same way that in [22, Lemma 4.3.8] we can prove that for each rational number h we have:

$$H^d_c(Y_K, \mathcal{L}(V^\vee_\lambda))^{\leqslant h} \xrightarrow{\sim} H^d_c(Y_K, \mathcal{L}(\mathbb{V}^\vee_\lambda))^{\leqslant h} \tag{5.1}$$

We fix $\sigma \in \Sigma_F$ and denote by λ_σ the algebraic character of T defined by:

$$\lambda(t) = a_\sigma^{\frac{-k_\sigma+r}{2}} b_\sigma^{\frac{k_\sigma+r}{2}} \prod_{\rho\in\Sigma_F-\sigma} a_\rho^{\frac{k_\rho-2+r}{2}} b_\rho^{\frac{-k_\rho+2+r}{2}},$$

for each $t = \left(\left(\begin{smallmatrix} a_\rho & 0 \\ 0 & b_\rho \end{smallmatrix}\right)\right)_{\rho\in\Sigma_F}$. From [22, Proposition 3.2.11] we have an I-equivariant morphism $\Theta_\sigma : \mathcal{A}_\lambda \to \mathcal{A}_{\lambda_\sigma}$, remark that this morphism is not equivariant with respect to the action of all Λ_p, in fact we have:

$$\Theta\left(\begin{pmatrix}1 & 0\\ 0 & p\end{pmatrix} * f\right) = p^{-k_\sigma+1}\begin{pmatrix}1 & 0\\ 0 & p\end{pmatrix} * \Theta(f).$$

We obtain an I-equivariant morphism:

$$\Theta_\sigma^\vee : \mathcal{D}_{\lambda_\sigma} \longrightarrow \mathcal{D}_\lambda. \tag{5.2}$$

We denote $\Sigma = \sum_{\sigma\in\Sigma_F}\Theta_\sigma^\vee$, then from [22, Proposition 3.2.12] we have the following exact sequence:

$$\bigoplus\nolimits_{\sigma\in\Sigma_F}\mathcal{D}_{\lambda_\sigma} \xrightarrow{\ \Sigma\ } \mathcal{D}_\lambda \longrightarrow V_\lambda^\vee \longrightarrow 0, \tag{5.3}$$

In fact, this sequence is the last part of the locally analytic BGG-resolution of $V_\lambda^\vee$ (see [22, §3.3]).

5.2. Proof of Theorem 5.1

From the discussion in 5.1 we obtain a morphism $H_c^d(Y_K, \mathcal{L}(\mathcal{D}_\lambda)) \to H_c^d(Y_K, \mathcal{L}(V_\lambda^\vee))$, this morphism is compatible with the Hecke operators. From (5.1) is enough to prove that $H_c^d(Y_K, \mathcal{L}(\mathcal{D}_\lambda))^{\leqslant h} \to H_c^d(Y_K, \mathcal{L}(V_\lambda^\vee))^{\leqslant h}$ is an isomorphism. We write $\Sigma_F = \{\sigma_1, \dots, \sigma_d\}$. We denote $\Sigma_0 = \emptyset$ and for each $s\in\{1,\dots,d\}$ we write:

$$\Sigma_s := \sum_{j=1}^{s}\Theta_{\sigma_j}^\vee : \bigoplus_{j=1}^{s}\mathcal{D}_{\lambda_{\sigma_j}} \to \mathcal{D}_\lambda.$$

For each $s\in\{1,\dots,d\}$ let Q_s be the quotient of $\mathcal{D}_{\lambda_{\sigma_s}}$ such that the following sequence is exact:

$$0 \longrightarrow Q_s \xrightarrow{\Theta_{\sigma_j}^\vee} \operatorname{coker}(\Sigma_{s-1}) \longrightarrow \operatorname{coker}(\Sigma_s) \longrightarrow 0.$$

Remark that Λ_p acts on $\operatorname{coker}(\Sigma_s)$ and Q_s, and in fact the last sequence is I-equivariant. However, this sequence is not Λ_p-equivariant, in fact we have:

$$\begin{tikzcd}
0 \arrow[r] & Q_s \arrow[r] \arrow[d, "*\left(\begin{smallmatrix}1&0\\0&p\end{smallmatrix}\right)p^{k_{\sigma_s}-1}"] & \operatorname{coker}(\Sigma_{s-1}) \arrow[r] \arrow[d, "*\left(\begin{smallmatrix}1&0\\0&p\end{smallmatrix}\right)"] & \operatorname{coker}(\Sigma_s) \arrow[r] \arrow[d, "*\left(\begin{smallmatrix}1&0\\0&p\end{smallmatrix}\right)"] & 0 \\
0 \arrow[r] & Q_s \arrow[r] & \operatorname{coker}(\Sigma_{s-1}) \arrow[r] & \operatorname{coker}(\Sigma_s) \arrow[r] & 0
\end{tikzcd}$$

Passing to the cohomology and by considering the action of U_p we obtain the following exact sequence:

$$H^d_c(Y_K,\mathcal{L}(Q_s))^{\leqslant h-(k_{\sigma_s}-1)} \longrightarrow H^d_c(Y_K,\mathcal{L}(\mathrm{coker}(\Sigma_{s-1})))^{\leqslant h}$$
$$\longrightarrow H^d_c(Y_K,\mathcal{L}(\mathrm{coker}(\Sigma_s)))^{\leqslant h} \longrightarrow H^{d+1}_c(Y_K,\mathcal{L}(Q_s))^{\leqslant h-(k_{\sigma_s}-1)}.$$

From 4.2.3 we deduce that in $H^i_c(Y_K,\mathcal{L}(Q_s))^{\leqslant h-(k_{\sigma_s}-1)}$ there is a O_L-lattice invariant by U_p, for any i. By assumption, for each s we have $h-(k_{\sigma_s}-1)<0$ then $H^i_c(Y_K,\mathcal{L}(Q_s))^{\leqslant h-(k_{\sigma_s}-1)}=\{0\}$, and so:

$$H^d_c(Y_K,\mathcal{L}(\mathrm{coker}(\Sigma_{s-1})))^{\leqslant h} \simeq H^d_c(Y_K,\mathcal{L}(\mathrm{coker}(\Sigma_s)))^{\leqslant h}.$$

Finally, clearly we have $\mathrm{coker}(\Sigma_0)=\mathcal{D}_\lambda$ and from (5.3) we obtain $\mathrm{coker}(\Sigma_d)=V_\lambda^\vee$ then we deduce the theorem.

6. Evaluations on the cohomology

In this section, to each class in the overconvergent cohomology we attach a distribution over the Galois group introduced in 4.3. Moreover we prove the admissibility of this distribution when the class is an eigenvector of U_p with slope non zero. To do that we use the automorphic cycles introduced in [9]. In all this section we fix an open compact subgroup of $G(\mathbb{A}_f)$, denoted by K, such that $\{\left(\begin{smallmatrix} u & v\\ 0 & 1\end{smallmatrix}\right) \mid u\in\hat{\mathcal{O}}_F^\times, v\in\hat{\mathcal{O}}_F\}\subset K$ and its image in $G(\mathbb{Q}_p)$ is contained in Λ_p. We fix $\{a_{\mathbf{x}}\mid \mathbf{x}\in\mathrm{Cl}_F^+\}\subset\mathbb{A}_{F,f}^\times$ a set of representatives of Cl_F^+ such that $a_{\mathbf{x}}^{-1}\in\hat{\mathcal{O}}_F$ and $a_{\mathbf{x},p}=1$. Using the notations of 2.2.1 we consider $g_{\mathbf{x}}$ to be $\left(\begin{smallmatrix} a_{\mathbf{x}} & 0\\ 0 & 1\end{smallmatrix}\right.$ for each $\mathbf{x}\in\mathrm{Cl}_F^+$.

6.1. Automorphic cycles

For $n\in\mathbb{N}$ we denote:

$$U(p^n):=\{u\in\hat{\mathcal{O}}_F^\times \mid u-1\in p^n\hat{\mathcal{O}}_F\} \text{ and } \mathrm{Cl}_F^+(p^n):=F^\times\setminus\mathbb{A}_F^\times/U(p^n)F_\infty^+.$$

The group $U(p^n)$ is open and compact in $\mathbb{A}_{F,f}^\times$. Let $E(p^n)=\mathcal{O}_F^\times\bigcap U(p^n)F_\infty^+$, then $E(1)$ is the group of totally positive units of $\mathcal{O}_F$. The real analytic variety $X_n:=F^\times\backslash\mathbb{A}_F^\times/U(p^n)$ has dimension d. For each $\mathbf{y}\in\mathrm{Cl}_F^+(p^n)$ we fix a representative $a_{\mathbf{y}}\in\mathbb{A}_F^\times$ we have the following decomposition in connected components:

$$X_n=\bigsqcup_{\mathbf{y}\in\mathrm{Cl}_F^+(p^n)} X_{n,\mathbf{y}},$$

where $X_{n,\mathbf{y}} = F^\times \setminus F^\times a_{\mathbf{y}} U(p^n) F_\infty^+ / U(p^n)$. The morphism $E(p^n) \setminus F_\infty^+ \to X_{n,\mathbf{y}}$ given by $[z] \to [a_{\mathbf{y}} z]$ is an analytic isomorphism. Moreover from the Dirichlet's Theorem $E(p^n) \setminus F_\infty^+$ is isomorphic to $(\mathbb{R}/\mathbb{Z})^{d-1} \times \mathbb{R}$. We deduce that $X_{n,\mathbf{y}}$ is connected and orientable.

If $x \in \mathbb{A}_F$ then we denote x_p by its image in $F \otimes \mathbb{Q}_p$. The morphism $\mathbb{A}_F^\times \to G(\mathbb{A})$, $x \mapsto \left(\begin{smallmatrix} x & x_p p^{-n} \\ 0 & 1 \end{smallmatrix}\right)$ induces a morphism of analytic varieties, called *automorphic cycle* in [9]: $C_{K,n} : X_n \to Y_K$.

6.2. Evaluations

We define certain evaluations on the overconvergent cohomology, these evaluations will be useful to construct our p-adic L-function and to prove its properties. Let $n \in \mathbb{N}$, we define the evaluations in four steps:

Step 1. — The cycle $C_{K,n}$ gives the morphism:

$$H_c^d(Y_K, \mathcal{L}(\mathcal{D}_\lambda)) \longrightarrow H_c^d(X_n, \mathcal{F}_n). \tag{6.1}$$

where $\mathcal{F}_n := C_{K,n}^*(\mathcal{L}(\mathcal{D}_\lambda))$. We can verify that $\mathcal{F}_n$ is the sheaf of locally constant sections of the local system:

$$F_n := F^\times \setminus (\mathbb{A}_F^\times \times \mathcal{D}_\lambda)/U(p^n) \longrightarrow X_n,$$

where the action on $\mathbb{A}_F^\times \times \mathcal{D}_\lambda$ is given by $f(x,\mu)v = (fxv, \mu * \left(\begin{smallmatrix} v & (v_p - 1)p^{-n} \\ 0 & 1 \end{smallmatrix}\right))$, $f \in F^\times, x \in \mathbb{A}_F^\times, \mu \in \mathcal{D}_\lambda$ and $v \in U(p^n)$.

Step 2. — Let $\mathcal{L}_n(\mathcal{D}_\lambda)$ be the sheaf over X_n given by the locally constant sections of the local system:

$$L_n(\mathcal{D}_\lambda) := F^\times \setminus (\mathbb{A}_F^\times \times \mathcal{D}_\lambda)/U(p^n) \longrightarrow X_n,$$

where $f(x,\mu)v = (fxv, \mu * \left(\begin{smallmatrix} v & 0 \\ 0 & 1 \end{smallmatrix}\right))$, $f \in F^\times, x \in \mathbb{A}_F^\times, \mu \in \mathcal{D}_\lambda$ and $v \in U(p^n)$.

The matrix $\left(\begin{smallmatrix} 1 & -1 \\ 0 & p^n \end{smallmatrix}\right) \in \Lambda_p$ satisfy $\left(\begin{smallmatrix} v & (v_p - 1)p^{-n} \\ 0 & 1 \end{smallmatrix}\right)\left(\begin{smallmatrix} 1 & -1 \\ 0 & p^n \end{smallmatrix}\right) = \left(\begin{smallmatrix} 1 & -1 \\ 0 & p^n \end{smallmatrix}\right)\left(\begin{smallmatrix} v & 0 \\ 0 & 1 \end{smallmatrix}\right)$ for each v. Then the morphism $\mathbb{A}_F^\times \times \mathcal{D}_\lambda \to \mathbb{A}_F^\times \times \mathcal{D}_\lambda$ given by $(x,\mu) \to (x, \mu * \left(\begin{smallmatrix} 1 & -1 \\ 0 & p^n \end{smallmatrix}\right))$ defines a morphism of sheaves $\mathcal{F}_n \to \mathcal{L}_n(\mathcal{D}_\lambda)$, then we obtain:

$$H_c^d(X_n, \mathcal{F}_n) \longrightarrow H_c^d(X_n, \mathcal{L}_n(\mathcal{D}_\lambda)). \tag{6.2}$$

Step 3. — Let $\mathcal{L}(\mathcal{D}_\lambda)$ be the sheaf over $E(p^n) \setminus F_\infty^+$ of locally constant sections of the local system:

$$E(p^n) \setminus (F_\infty^+ \times \mathcal{D}_\lambda) \longrightarrow E(p^n) \setminus F_\infty^+,$$

where $e(c,\mu) = (ce, \mu * \left(\begin{smallmatrix} e & 0 \\ 0 & 1 \end{smallmatrix}\right))$ for $e \in E(p^n)$, $c \in F_\infty^+$ and $\mu \in \mathcal{D}_\lambda$.

Let $\mathbf{y} \in \mathrm{Cl}_F^+(p^n)$ and let $h : E(p^n) \setminus F_\infty^+ \to X_{n,\mathbf{y}}$ be given by the representative, $a_\mathbf{y}$, of $\mathbf{y}$. We have $h^\times(\mathcal{L}_n(\mathcal{D}_\lambda)|_{X_{n,\mathbf{y}}}) = \mathcal{L}(\mathcal{D}_\lambda)$ and then we obtain an isomorphism:

$$H_c^d(X_{n,\mathbf{y}}, \mathcal{L}_n(\mathcal{D}_\lambda)|_{X_{n,\mathbf{y}}}) \simeq H_c^d(E(p^n) \setminus F_\infty^+, \mathcal{L}(\mathcal{D}_\lambda)). \tag{6.3}$$

The function $\mathcal{D}_\lambda \to \mathcal{D}_\lambda^+$ defined by $\mu \to \mu\,|_{\mathcal{A}_\lambda^+}$ (see 4.2.5) gives a morphism of sheaves over $E(p^n) \setminus F_\infty^+$: $\mathcal{L}(\mathcal{D}_\lambda)) \to \mathcal{D}_\lambda^+$. Here, by abuse of notation, $\mathcal{D}_\lambda^+$ means the constant sheaf with stalk $\mathcal{D}_\lambda^+$. We obtain a morphism:

$$H_c^d(E(p^n) \setminus F_\infty^+, \mathcal{L}(\mathcal{D}_\lambda)) \longrightarrow H_c^d(E(p^n) \setminus F_\infty^+, \mathcal{D}_\lambda^+). \tag{6.4}$$

The real analytic variety $E(p^n) \setminus F_\infty^+$ is connected, orientable and of dimension d then $H_c^d(E(p^n) \setminus F_\infty^+, \mathcal{D}_\lambda^+) \simeq \mathcal{D}_\lambda^+$. From (6.3) and (6.4) we obtain:

$$H_c^d(X_n, \mathcal{L}_n(\mathcal{D}_\lambda)) \longrightarrow (\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^n)}. \tag{6.5}$$

Step 4. — Finally from (6.1), (6.2) and (6.5). we get the following evaluation:

$$\mathbf{ev}_{K,n} : H_c^d(Y_K, \mathcal{L}(\mathcal{D}_\lambda)) \longrightarrow (\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^n)}.$$

These different evaluations are related by:

Lemma 6.1. — *For each $n \geqslant 1$ we have the following commutative diagram:*

$$\begin{tikzcd} H_c^d(Y_K, \mathcal{L}_K(\mathcal{D}_\lambda)) \arrow[r, "U_p"] \arrow[d, "\mathbf{ev}_{K,n+1}"] & H_c^d(Y_K, \mathcal{L}_K(\mathcal{D}_\lambda)) \arrow[d, "\mathbf{ev}_{K,n}"] \\ (\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^{n+1})} \arrow[r, "tr_n"] & (\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^n)} \end{tikzcd}$$

here $tr_n : (\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^{n+1})} \to (\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^n)}$ is the morphism

$$tr_n((\mu_\mathbf{x})_{\mathbf{x}\in\mathrm{Cl}_F^+(p^{n+1})}) = (\nu_\mathbf{y})_{\mathbf{y}\in\mathrm{Cl}_F^+(p^n)}$$

with $\nu_\mathbf{y} = \sum_{\mathbf{x}\to\mathbf{y}} \mu_\mathbf{x}$, where $\mathbf{x} \to \mathbf{y}$ means the set of $\mathbf{x} \in \mathrm{Cl}_F^+(p^{n+1})$ whose image in $\mathrm{Cl}_F^+(p^n)$ is $\mathbf{y}$.

Proof. — We prove that in each step of the construction of our evaluations we have a commutative diagram. Firstly we construct a morphism $H_c^d(X_{n+1}, \mathcal{F}_{n+1}) \to H_c^d(X_n, \mathcal{F}_n)$. Let $pr_n : X_{n+1} \to X_n$ be the canonical morphism, $\mathcal{F} := (pr_n)_*(\mathcal{F}_{n+1})$ and $\mathfrak{G} = \mathrm{Gal}(X_{n+1}/X_n) \simeq \mathcal{O}_F/p\mathcal{O}_F$. The morphism $\alpha : F_{n+1} \to F_n$ given by $(y, \mu) \to (y, \mu * \left(\begin{smallmatrix} 1 & 0 \\ 0 & p \end{smallmatrix}\right))$ is well defined because we have the following identity:

$$\begin{pmatrix} v & (v_p - 1)p^{-n-1} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix} \begin{pmatrix} v & (v_p - 1)p^{-n} \\ 0 & 1 \end{pmatrix}.$$

Let $U \subset X_n$ be an open small enough such that $pr_n^{-1}(U) = \sqcup_{g\in\mathfrak{G}} U_g \subset X_{n+1}$ and for each g pr_n induces an homeomorphism $i_g : U \to U_g$, then we have $\Gamma(U,\mathcal{F}) = \Gamma(\mathrm{pr}^{-1}(U), \mathcal{F}_{n+1}) = \oplus_{g\in\mathfrak{G}} \Gamma(U_g, \mathcal{F}_{n+1})$. We define:

$$\Gamma(U,\mathcal{F}) \longrightarrow \Gamma(U,\mathcal{F}_n), \qquad s = (s_g)_{g\in\mathfrak{G}} \longrightarrow \textstyle\sum_{g\in\mathfrak{G}} \alpha \circ s_g \circ i_g.$$

We obtain a morphism of sheaves $\mathcal{F} \to \mathcal{F}_n$. Remark that $H_c^d(X_{n+1}, \mathcal{F}_{n+1}) = H_c^d(X_n, \mathcal{F})$, then we obtain:

$$H_c^d(X_{n+1}, \mathcal{F}_{n+1}) \xrightarrow{*\left(\begin{smallmatrix}1 & 0\\ 0 & p\end{smallmatrix}\right)} H_c^d(X_n, \mathcal{F}_n) .$$

We have the following commutative diagram:

$$\begin{tikzcd}
H_c^d(Y_K, \mathcal{L}_K(\mathcal{D}_\lambda)) \arrow[r, "U_p"] \arrow[d] & H_c^d(Y_K, \mathcal{L}_K(\mathcal{D}_\lambda)) \arrow[d] \\
H_c^d(X_{n+1}, \mathcal{F}_{n+1}) \arrow[r, "*\left(\begin{smallmatrix}1 & 0\\ 0 & p\end{smallmatrix}\right)"] & H_c^d(X_n, \mathcal{F}_n)
\end{tikzcd}$$

Denote $\mathcal{F}' = \mathrm{pr}_*(\mathcal{L}_{n+1}(\mathcal{D}_\lambda))$, if we repeat the last construction using instead of α the morphism $\alpha' : L_{n+1}(\mathcal{D}_\lambda) \to L_n(\mathcal{D}_\lambda)$ defined by $(y,\mu) \to (y,\mu)$, we obtain a morphism X_n: $\mathcal{F}' \to \mathcal{L}_n(\mathcal{D}_\lambda)$ of sheaves over X_n. This morphism gives us the *trace* morphism in the cohomology $H_c^d(X_{n+1}, \mathcal{L}_{n+1}(\mathcal{D}_\lambda)) \to H_c^d(X_n, \mathcal{L}_n(\mathcal{D}_\lambda))$. From $\left(\begin{smallmatrix}1 & 0\\ 0 & p\end{smallmatrix}\right)\left(\begin{smallmatrix}1 & -1\\ 0 & p^n\end{smallmatrix}\right) = \left(\begin{smallmatrix}1 & -1\\ 0 & p^{n+1}\end{smallmatrix}\right)$ we deduce the following commutative diagram:

$$\begin{tikzcd}
H_c^d(X_{n+1}, \mathcal{F}_{n+1}) \arrow[r, "*\left(\begin{smallmatrix}1 & 0\\ 0 & p\end{smallmatrix}\right)"] \arrow[d] & H_c^d(X_n, \mathcal{F}_n) \arrow[d] \\
H_c^d(X_{n+1}, \mathcal{L}_{n+1}(\mathcal{D}_\lambda)) \arrow[r, "trace"] & H_c^d(X_n, \mathcal{L}_n(\mathcal{D}_\lambda))
\end{tikzcd}$$

Finally decomposing the morphism $H_c^d(X_{n+1}, \mathcal{L}_{n+1}(\mathcal{D}_\lambda)) \to H_c^d(X_n, \mathcal{L}_n(\mathcal{D}_\lambda))$ on the connected components of X_n and X_{n+1} we obtain the following commutative diagram:

$$\begin{tikzcd}
H_c^d(X_{n+1}, \mathcal{L}_{n+1}(\mathcal{D}_\lambda)) \arrow[r, "trace"] \arrow[d] & H_c^d(X_n, \mathcal{L}_n(\mathcal{D}_\lambda)) \arrow[d] \\
(\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^{n+1})} \arrow[r, "tr_n"] & (\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^n)}
\end{tikzcd}$$

□

6.3. Construction

Using $\mathbf{ev}_{K,1}$ we achieve to attach a distribution to each class in the overconvergent cohomology.

Let $\Phi \in H^d_c(Y_K, \mathcal{L}(\mathcal{D}_\lambda))$ and we write

$$\mathbf{ev}_{K,1}(\Phi) = (\nu_{\mathbf{y}})_{\mathbf{y}\in \mathrm{Cl}^+_F(p)} \in (\mathcal{D}^+_\lambda)^{\mathrm{Cl}^+_F(p)}.$$

For each $\mathbf{x} \in \mathrm{Cl}^+_F$ we define:

$$\mu_{\mathbf{x}} = \sum_{\mathbf{y}\in \mathrm{Cl}^+_F(p)_{\mathbf{x}}} (\nu_{\mathbf{y}})^\times \in \mathcal{D}(\mathrm{Gal}^\circ_p, L),$$

here $\mathrm{Cl}^+_F(p)_{\mathbf{x}}$ is the set of $\mathbf{y} \in \mathrm{Cl}^+_F(p)$ whose image in Cl^+_F is $\mathbf{x}$, and $(\nu_{\mathbf{y}})^\times$ is the image of $\nu_{\mathbf{y}}$ under $\mathcal{D}^+_\lambda \to \mathcal{D}(\mathrm{Gal}^\circ_p, L)$. Finally let $\mu_\Phi \in \mathcal{D}(\mathrm{Gal}_p, L)$ be the distribution corresponding to $(\mu_{\mathbf{x}})_{\mathbf{x}\in\mathrm{Cl}^+_F} \in \mathcal{D}(\mathrm{Gal}^\circ_p, L)^{\mathrm{Cl}^+_F}$ using the isomorphism (4.3). Then we have:

$$\begin{aligned} \mu_\Phi : \mathcal{A}(\mathrm{Gal}_p, L) &\longrightarrow L, \\ f &\longmapsto \sum_{\mathbf{x}\in\mathrm{Cl}^+_F} \mu_{\mathbf{x}}(f_{\mathbf{x}}) \end{aligned} \tag{6.6}$$

where $f_{\mathbf{x}} \in \mathcal{A}(\mathrm{Gal}^\circ_p, L)$ is defined in 4.3.2.

The following diagram summarize our construction:

$$\begin{tikzcd} H^d_c(Y_K, \mathcal{L}(\mathcal{D}_\lambda)) \arrow[r, "\mathbf{ev}_{K,1}"] & (\mathcal{D}^+_\lambda)^{\mathrm{Cl}^+_F(p)} \arrow[r] & (\mathcal{D}^+_\lambda)^{\mathrm{Cl}^+_F} \arrow[d] \\ & \mathcal{D}(\mathrm{Gal}_p, L) & \mathcal{D}(\mathrm{Gal}^\circ_p, L)^{\mathrm{Cl}^+_F} \arrow[l, "\sim"'] \end{tikzcd}$$

6.4. Classical cycles and evaluations

The results of this subsection will be used to prove that the distribution $\mu_\Phi \in \mathcal{D}(\mathrm{G}_p, L)$ is admissible (see 6.5).

6.4.1. Cycles

Let $f \in F$ and $E \subset E(1)$ be a subgroup of finite index. Let $\Gamma \subset \mathrm{GL}_2(F)$ be an arithmetic subgroup such that $\left(\begin{smallmatrix} e & (1-e)f \\ 0 & 1 \end{smallmatrix}\right) \in \Gamma$ for all $e \in E$. Then $y \to f + iy$ induces a morphism:

$$c_f : E\backslash F^+_\infty \longrightarrow \Gamma\backslash \mathbb{H}_F.$$

Remark 6.2. — Let $f, f' \in F$ be such that $\left(\begin{smallmatrix} 1 & (f-f') \\ 0 & 1 \end{smallmatrix}\right) \in \Gamma$ then $c_f = c_{f'}$.

Let $\pi_n : \mathrm{Cl}_F^+(p^n) \to \mathrm{Cl}_F^+$ be the natural map. For each $\mathbf{x} \in \mathrm{Cl}_F^+$ we write $\mathrm{Cl}_F^+(p^n)_{\mathbf{x}} := \pi_n^{-1}(\mathbf{x})$. Recall that from hypothesis on K we deduce that Y_K has $\sharp\,\mathrm{Cl}_F^+$ connected components, $Y_K = \bigsqcup_{\mathbf{x}\in \mathrm{Cl}_F^+} Y_{\mathbf{x}}$.

Lemma 6.3. — *We have:*

(1) *If* $\mathbf{x} \in \mathrm{Cl}_F^+$ *then* $\mathrm{Cl}_F^+(p^n)_{\mathbf{x}} = F^\times \setminus F^\times a_{\mathbf{x}} F_\infty^+ \hat{\mathcal{O}}_F^\times / U(p^n) F_\infty^+$ *and we have an isomorphism* $\frac{\hat{\mathcal{O}}_F^\times}{U(p^n)E(1)} \simeq \mathrm{Cl}_F^+(p^n)_{\mathbf{x}}$. *Moreover the association* $\mathcal{O}_F \to \hat{\mathcal{O}}_F$ *given by* $a \to u_a$, *where* u_a *is* 1 *at the premier numbers different that* p *and* a *in* p, *gives us:*

$$\frac{\left(\frac{\mathcal{O}_F}{p^n\mathcal{O}_F}\right)^\times}{E(1)} \xrightarrow{\sim} \frac{\hat{\mathcal{O}}_F^\times}{U(p^n)E(1)} \xrightarrow{\sim} \mathrm{Cl}_F^+(p^n)_{\mathbf{x}}$$

(2) *Fix a set of representatives* $S_n \subset \mathcal{O}_F$ *for* $\frac{\left(\frac{\mathcal{O}_F}{p^n\mathcal{O}_F}\right)^\times}{E(1)}$. *Then we have the following commutative diagram:*

$$\begin{tikzcd}
\bigsqcup_{a\in S_n} E(p^n)\setminus F_\infty^+ \arrow[r, "\sqcup_a c_{-a\backslash p^{-n}}"] \arrow[d, "\sim"] & \Gamma_{\mathbf{x}} \setminus \mathbb{H}_F \arrow[d, "\sim"] \\
\sqcup_{\mathbf{y}\in \mathrm{Cl}_F^+(p^n)_{\mathbf{x}}} X_{n,\mathbf{y}} \arrow[r, "C_{K,n}"] & Y_{\mathbf{x}}
\end{tikzcd}$$

Proof. — The part (1) is clear. Let $a \in S_n$ and call $\mathbf{y}$ the corresponding class in $\mathrm{Cl}_F^+(p^n)_{\mathbf{x}}$. Let $[r] \in E(p^n) \setminus F_\infty^+$ then its image in $X_{n,\mathbf{y}}$ is $[r a_{\mathbf{x}} u_a]$ and we have $C_{K,n}([r a_{\mathbf{x}} u_a]) = [\left(\begin{smallmatrix} r a_{\mathbf{x}} u_a & (ap^{-n})_p \\ 0 & 1 \end{smallmatrix}\right)] = [\left(\begin{smallmatrix} r a_{\mathbf{x}} & (-ap^{-n})_\infty \\ 0 & 1 \end{smallmatrix}\right)]$, the last identity comes from:

$$\begin{pmatrix} r a_{\mathbf{x}} u_a & (ap^{-n})_p \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & ap^{-n} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} r a_{\mathbf{x}} & (-ap^{-n})_\infty \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_a & (ap^{-n} a_{\mathbf{x}}^{-1})_f^p \\ 0 & 1 \end{pmatrix},$$

here $(ap^{-n}a_{\mathbf{x}}^{-1})_f^p$ denotes the finite adele which is $(ap^{-n}a_{\mathbf{x}}^{-1})_l$ for $l \neq p$ and 0 for $l = p$. Finally $c_{-a\backslash p^{-n}}([r]) = [-\frac{a}{p^n} + ir] = [\left(\begin{smallmatrix} r & -ap^{-n} \\ 0 & 1 \end{smallmatrix}\right) i]$ then its image in $Y_{\mathbf{x}}$ is $[\left(\begin{smallmatrix} r & (-ap^{-n})_\infty \\ 0 & 1 \end{smallmatrix}\right) g_{\mathbf{x}}] = [\left(\begin{smallmatrix} r & (-ap^{-n})_\infty \\ 0 & 1 \end{smallmatrix}\right)\left(\begin{smallmatrix} a_{\mathbf{x}} & 0 \\ 0 & 1 \end{smallmatrix}\right)] = C_{K,n}([r a_{\mathbf{x}} u_a])$. □

6.4.2. Evaluations

We define evaluations on the cohomology of $\Gamma \setminus \mathbb{H}_F$ in the same way that in 6.2. Let $f \in F$, $E \subset E(1)$ and $\Gamma \subset \mathrm{GL}_2(F)$ be as before, moreover we suppose that the image of Γ in $G(\mathbb{Q}_p)$ is contained in Λ_p and fix a decomposition $f = ab^{-1}$, where $a, b \in \mathcal{O}_F$ and $b \neq 0$.

Step 1. — Using c_f instead of $C_{K,n}$ in step 1 of 6.2 we obtain:

$$H_c^d(\Gamma\setminus\mathbb{H}_F,\mathcal{L}(\mathcal{D}_\lambda))\xrightarrow{c_f^*}H_c^d(E\setminus F_\infty^+,c_f^\times\mathcal{L}(\mathcal{D}_\lambda))\ .$$

Step 2. — Explicitly the sheaf $c_f^*\mathcal{L}(\mathcal{D}_\lambda)$ is given by the fiber bundle $E\setminus(F_\infty^+\times\mathcal{D}_\lambda)\to E\setminus F_\infty^+$ where $e(y,\mu)=(ey,\mu*\left(\begin{smallmatrix}e^{-1}&f(1-e^{-1})\\0&1\end{smallmatrix}\right))$. So using $\left(\begin{smallmatrix}1&a\\0&b\end{smallmatrix}\right)$ instead $\left(\begin{smallmatrix}1&-1\\0&p^n\end{smallmatrix}\right)$ of step 2 in 6.2 we obtain:

$$H_c^d(E\setminus F_\infty^+,c_f^*\mathcal{L}(\mathcal{D}_\lambda))\xrightarrow{*\left(\begin{smallmatrix}1&a\\0&b\end{smallmatrix}\right)}H_c^d(E\setminus F_\infty^+,\mathcal{L}(\mathcal{D}_\lambda)),$$

here $\mathcal{L}(\mathcal{D}_\lambda)$ is the sheaf over $E\setminus F_\infty^+$ given by $F_\infty^+\times\mathcal{D}_\lambda$ where the action is $e(y,\mu)=(ey,\mu*\left(\begin{smallmatrix}e^{-1}&0\\0&1\end{smallmatrix}\right))$.

Step 3. — Finally, using $H_c^d(E\setminus F_\infty^+,\mathcal{L}(\mathcal{D}_\lambda)\to H_c^d(E\setminus F_\infty^+,\mathcal{D}_\lambda^+)\simeq\mathcal{D}_\lambda^+$ we obtain:

$$\mathrm{ev}_{\Gamma,f}:H_c^d(\Gamma\setminus\mathbb{H}_F,\mathcal{L}(\mathcal{D}_\lambda))\longrightarrow\mathcal{D}_\lambda^+.$$

6.4.3. Description of $\mathrm{ev}_{\Gamma,f}$

Let $\mathcal{A}_\lambda^f$ be the subspace of $\mathcal{A}_\lambda(L)$ of functions g such that $\left(\begin{smallmatrix}e^{-1}&f(1-e^{-1})\\0&1\end{smallmatrix}\right)*g=g$ for all $e\in E$. This space is a Frechet space and we denote $\mathcal{D}_\lambda^f(L)=\mathcal{D}_\lambda^f$ its continuous dual. The morphism $\mathcal{A}_\lambda^+\to\mathcal{A}_\lambda^f:\ g\to\left(\begin{smallmatrix}1&a\\0&b\end{smallmatrix}\right)*g$ is continuous and then gives us $\mathcal{D}_\lambda^f\to\mathcal{D}_\lambda^+$.As in 4 we consider the subspace $\mathcal{A}_{\lambda,0}^f\subset\mathcal{A}_\lambda^f$ of functions that can be expressed as a converging power series on all $\mathcal{O}_F\otimes\mathbb{Z}_p$. This space is a Banach space and we call $\mathcal{D}_{\lambda,0}^f$ its dual. We have the restriction $\mathcal{D}_{\lambda,0}\to\mathcal{D}_{\lambda,0}^f$. In the same way, we can define spaces $\mathcal{A}_{\lambda,0}^f(\mathcal{O}_L)$ and $\mathcal{D}_{\lambda,0}^f(\mathcal{O}_L)$ and we have the restriction morphism $\mathcal{D}_{\lambda,0}(\mathcal{O}_L)\to\mathcal{D}_{\lambda,0}^f(\mathcal{O}_L)$.

Remark 6.4. — Let $a',a\in\mathcal{O}_F$ be defining the same class in $\frac{\left(\frac{\mathcal{O}_F}{p^n\mathcal{O}_F}\right)^\times}{E(1)}$, is not difficult to obtain an isomorphism between $\mathcal{D}_\lambda^{ap^{-n}}$ and $\mathcal{D}_\lambda^{a'p^{-n}}$, and moreover the following diagram is commutative:

$$\begin{tikzcd}
\mathcal{D}_\lambda^{ap^{-n}} \arrow[r] \arrow[d, "\sim"] & \mathcal{D}_\lambda^+ \\
\mathcal{D}_\lambda^{a'p^{-n}} \arrow[ur] &
\end{tikzcd}$$

In particular the image of $\mathcal{D}_\lambda^{ap^{-n}} \to \mathcal{D}_\lambda^+$ depend only on the class of a in $\frac{\left(\frac{\mathcal{O}_F}{p^n\mathcal{O}_F}\right)^\times}{E(1)}$.

Remark 6.5. — The restriction morphism $\mathcal{D}_\lambda \to \mathcal{D}_\lambda^f$ gives $H_c^d(E\backslash F_\infty^+, c_f^*\mathcal{L}(\mathcal{D}_\lambda)) \to H_c^d(E\backslash F_\infty^+, \mathcal{D}_\lambda^f) \simeq \mathcal{D}_\lambda^f$. So we obtain a morphism $H_c^d(\Gamma\backslash\mathbb{H}_F, \mathcal{L}(\mathcal{D}_\lambda)) \to \mathcal{D}_\lambda^f$. Moreover by definition we have the following commutative diagram:

$$\begin{tikzcd} \mathrm{ev}_{\Gamma,f} : H_c^d(\Gamma\setminus\mathbb{H}_F, \mathcal{L}(\mathcal{D}_\lambda)) \arrow[r, "c_f^*"] & H_c^d(E\setminus F_\infty^+, c_f^*\mathcal{L}(\mathcal{D}_\lambda)) \arrow[r] \arrow[d] & \mathcal{D}_\lambda^+ \\ & \mathcal{D}_\lambda^f \arrow[ur, "*\left(\begin{smallmatrix}1 & a\\ 0 & b\end{smallmatrix}\right)"'] & \end{tikzcd} \tag{6.7}$$

Remark 6.6. — In the same way that in the last remark we can define morphisms:

$$H_c^d(\Gamma\backslash\mathbb{H}_F, \mathcal{L}(\mathcal{D}_{\lambda,0})) \to \mathcal{D}_{\lambda,0}^f, \quad \text{and} \quad H_c^d(\Gamma\backslash\mathbb{H}_F, \mathcal{L}(\mathcal{D}_{\lambda,0}(\mathcal{O}_L))) \to \mathcal{D}_{\lambda,0}^f(\mathcal{O}_L).$$

6.4.4. $\mathcal{O}_L$-modules

LEMMA 6.7. — *Let $\Gamma \subset G(\mathbb{Q})$ be an arithmetic group with image in $G(\mathbb{Q}_p)$ contained in Λ_p. Let $\phi \in H_c^d(\Gamma\backslash\mathbb{H}_F, \mathcal{L}(\mathcal{D}_\lambda))$, then there exist $C(\phi) > 0$ such that:*

- *if (f, E) is a pair satisfying conditions in 6.4.2 to respect with Γ and if $\nu \in \mathcal{D}_\lambda^f$ is the image of ϕ under the morphism $H_c^d(\Gamma \setminus \mathbb{H}_F, \mathcal{L}(\mathcal{D}_\lambda)) \to \mathcal{D}_\lambda^f$ defined at the end of 6.4.2, then:*

$$\|\nu\|_0 \leqslant C(\phi).$$

Proof. — Considering the notation of 4, 6.4.3 and by definition we obtain the following commutative diagram:

$$\begin{tikzcd} H_c^d(\Gamma\backslash\mathbb{H}_F, \mathcal{L}(\mathcal{D}_\lambda)) \arrow[r, "c_f^*"] \arrow[d] & H_c^d(E\backslash F_\infty^+, c_f^*\mathcal{L}(\mathcal{D}_\lambda)) \arrow[r] \arrow[d] & \mathcal{D}_\lambda^f \arrow[d] \\ H_c^d(\Gamma\backslash\mathbb{H}_F, \mathcal{L}(\mathcal{D}_{\lambda,0})) \arrow[r, "c_f^*"] & H_c^d(E\backslash F_\infty^+, c_f^*\mathcal{L}(\mathcal{D}_{\lambda,0})) \arrow[r] & \mathcal{D}_{\lambda,0}^f \\ H_c^d(\Gamma\backslash\mathbb{H}_F, \mathcal{L}(\mathcal{D}_{\lambda,0}(\mathcal{O}_L))) \arrow[r, "c_f^*"] \arrow[u] & H_c^d(E\backslash F_\infty^+, c_f^*\mathcal{L}(\mathcal{D}_{\lambda,0}(\mathcal{O}_L))) \arrow[r] \arrow[u] & \mathcal{D}_{\lambda,0}^f(\mathcal{O}_L) \arrow[u] \end{tikzcd}$$

$\mathcal{D}_{\lambda,0}$ is a Banach L-space then there exists $\beta \in L^\times$ such that the image of $\beta\phi$ through $H_c^d(\Gamma \setminus \mathbb{H}_F, \mathcal{L}(\mathcal{D}_\lambda)) \to H_c^d(\Gamma \setminus \mathbb{H}_F, \mathcal{L}(\mathcal{D}_{\lambda,0}))$ is contained in $H_c^d(\Gamma \setminus \mathbb{H}_F, \mathcal{L}(\mathcal{D}_{\lambda,0}(\mathcal{O}_L)))$. We write $C(\phi) = \mid \beta \mid_p^{-1}$.

Let (f, E) be as before and let $\nu \in \mathcal{D}_\lambda^f$ be the image of ϕ under the morphism $H_c^d(\Gamma \setminus \mathbb{H}_F, \mathcal{L}(\mathcal{D}_\lambda)) \to \mathcal{D}_\lambda^f$. Because the choice of β the image of $\beta\nu$ in $\mathcal{D}_{\lambda,0}^f$ is in fact contained in $\mathcal{D}_{\lambda,0}^f(\mathcal{O}_L)$ and then $\|\beta\nu\|_0 \leqslant 1$. Then we obtain:

$$\|\nu\|_0 = C(\Phi)\|\beta\nu\|_0 \leqslant C(\Phi). \qquad \square$$

6.4.5. Classical and automorphic evaluations

Fix $\Phi \in H_c^d(Y_K, \mathcal{L}(\mathcal{D}_\lambda))$ and $n \geqslant 1$. We denote $\mathbf{ev}_{K,n}(\Phi) = (\nu_\mathbf{y})_{\mathbf{y} \in \mathrm{Cl}_F^+(p^n)} \in (\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^n)}$. Fix $\mathbf{x} \in \mathrm{Cl}_F^+$ and $a \in S_n$, then denote by $\nu_{\mathbf{x},a} \in \mathcal{D}_\lambda^{-ap^{-n}}$ the image of Φ under:

$$H_c^d(Y_K, \mathcal{L}(\mathcal{D}_\lambda)) \longrightarrow H_c^d(Y_\mathbf{x}, \mathcal{L}(\mathcal{D}_\lambda)) \longrightarrow H_c^d(\Gamma_\mathbf{x} \setminus \mathbb{H}_F, \mathcal{L}(\mathcal{D}_\lambda)) \longrightarrow \mathcal{D}_\lambda^{-ap^{-n}}$$

here the last morphism was described in the end of 6.4.2.

Lemma 6.8. — *Let $\mathbf{y} \in \mathrm{Cl}_F^+(p^n)_\mathbf{x}$ be the image of $a \in S_n$ under the bijection $\frac{\left(\frac{\mathcal{O}_F}{p^n\mathcal{O}_F}\right)^\times}{E(1)} \simeq \mathrm{Cl}_F^+(p^n)_\mathbf{x}$. Then we have:*

$$\nu_\mathbf{y} = \nu_{\mathbf{x},a} * \begin{pmatrix} 1 & -a \\ 0 & p^n \end{pmatrix}.$$

Proof. — From Lemma 6.3 we obtain the following commutative diagram:

$$\begin{tikzcd}
H_c^d(Y_\mathbf{x}, \mathcal{L}(\mathcal{D}_\lambda)) \arrow[r] \arrow[d, "C_n^*"] & H_c^d(\Gamma_\mathbf{x} \setminus \mathbb{H}_F, \mathcal{L}(\mathcal{D}_\lambda)) \arrow[d, "\oplus_{a\in S_n} c^*_{-ap^{-n}}"] \\
H_c^d(\sqcup_{\mathbf{y}\in \mathrm{Cl}_F^+(p^n)_\mathbf{x}} X_{n,\mathbf{y}}, C_n^*\mathcal{L}(\mathcal{D}_\lambda)) \arrow[r] \arrow[d, "*\left(\begin{smallmatrix} 1 & -1 \\ 0 & p^n \end{smallmatrix}\right)"] & \oplus_{a\in S_n} H_c^d(E(p^n) \setminus F_\infty^+, c^*_{-ap^{-n}}\mathcal{L}(\mathcal{D}_\lambda)) \arrow[d, "\oplus_{a\in S_n} *\left(\begin{smallmatrix} 1 & -a \\ 0 & p^n \end{smallmatrix}\right)"] \\
H_c^d(\sqcup_{\mathbf{y}\in \mathrm{Cl}_F^+(p^n)_\mathbf{x}} X_{n,\mathbf{y}}, \mathcal{L}_n(\mathcal{D}_\lambda)) \arrow[r] \arrow[dr] & H_c^d(E(p^n) \setminus F_\infty^+, \mathcal{L}_n(\mathcal{D}_\lambda))^{S_n} \arrow[d] \\
 & \mathcal{D}_\lambda^{\mathrm{Cl}_F^+(p^n)_\mathbf{x}}
\end{tikzcd}$$

Here the right column is by definition $(\mathrm{ev}_{\Gamma_\mathbf{x}, -ap^{-n}})_{a\in S_n}$. Finally we obtain the result from (6.7) $\square$

6.5. Admissibility

Lemma 6.9. — *Let $\Phi \in H^d_c(Y_K, \mathcal{L}(\mathcal{D}_\lambda))$. Then there exists $C = C(\Phi) > 0$, depending only on Φ, such that for each $n \geqslant 1$ we have $\|\nu_{\mathbf{y}}\|_n \leqslant C$ where $\mathbf{ev}_{K,n}(\Phi) = (\nu_{\mathbf{y}})_{\mathbf{y}\in \mathrm{Cl}^+_F(p^n)}$.*

Proof. — Let $\mathbf{x} \in \mathrm{Cl}^+_F$. Let $\Phi_{\mathbf{x}} \in H^d_c(\Gamma_{\mathbf{x}} \backslash \mathbb{H}_F, \mathcal{L}(\mathcal{D}_\lambda))$ be the image of Φ. Let $C_{\mathbf{x}} > 0$ be given by Lemma 6.7. If $n \geqslant 1$ and $\mathbf{y} \in \mathrm{Cl}^+_F(p^n)_{\mathbf{x}}$ let $a \in S_n$ be corresponding to $\mathbf{y}$ by the isomorphism $\frac{\left(\frac{\mathcal{O}_F}{p^n\mathcal{O}_F}\right)^\times}{E(1)} \xrightarrow{\sim} \mathrm{Cl}^+_F(p^n)_{\mathbf{x}}$. Then from 6.8 and 6.7 we have:

$$\|\nu_{\mathbf{x}}\|_n = \|\nu_{\mathbf{x},a} * \left(\begin{smallmatrix} 1 & -a \\ 0 & p^n \end{smallmatrix}\right)\|_n \leqslant \|\nu_{\mathbf{x},a}\|_0 \leqslant C_{\mathbf{x}}.$$

We take $C(\Phi) := \max\{C_{\mathbf{x}} \mid \mathbf{x} \in \mathrm{Cl}^+_F\}$ □

Proposition 6.10. — *Let $\Phi \in H^d_c(Y_K, \mathcal{L}(\mathcal{D}_\lambda))$ be such that $U_p(\Phi) = \alpha\Phi$ where $\alpha \in L^\times$. We denote $h = \mathrm{v}_p(\alpha)$ then $\mu_\Phi \in \mathcal{D}(\mathrm{Gal}_p, L)$ is an h-admissible distribution.*

Proof. — As $\mu_\Phi \in \mathcal{D}(\mathrm{Gal}_p, L)$ is obtained from $(\mu_{\mathbf{x}})_{\mathbf{x}\in \mathrm{Cl}^+_F} \in \mathcal{D}(\mathrm{Gal}^\circ_p, L)^{\mathrm{Cl}^+_F}$ under isomorphism 4.3, then it is enough to prove that $\mu_{\mathbf{x}}$ is h-admissible for each $\mathbf{x} \in \mathrm{Cl}^+_F$. Fix $\mathbf{x} \in \mathrm{Cl}^+_F$. Let $n \geqslant 1$ and denote $\mathbf{ev}_{K,n}(\Phi) = (\nu_{\mathbf{y}})_{\mathbf{y}\in \mathrm{Cl}^+_F(p^n)}$ then using Lemma 6.1 we obtain:

$$\mu_{\mathbf{x}} = \alpha^{-n+1} \sum_{\mathbf{y}\in \mathrm{Cl}^+_F(p^n)_{\mathbf{x}}} (\nu_{\mathbf{y}})^\times.$$

Let $C > 0$ be the constant obtained in 4.8 then $\|\mu_{\mathbf{x}}\|_n \leqslant$:

$$p^{nh-h}\max\{\|(\nu_{\mathbf{y}})^\times\|_n \mid \mathbf{y} \in \mathrm{Cl}^+_F(p^n)_{\mathbf{x}}\} \leqslant Cp^{nh-h}\max\{\|\nu_{\mathbf{y}}\|_n \mid \mathbf{y} \in \mathrm{Cl}^+_F(p^n)_{\mathbf{x}}\}$$

Let $C(\Phi) > 0$ be the constant obtained in 6.9, then we deduce that for each $n \geqslant 1$ we have $\|\mu_{\mathbf{x}}\|_n \leqslant C(\Phi)Cp^{-h}p^{nh}$, so $\mu_{\mathbf{x}}$ is h-admissible. □

7. Automorphic representations and p-adic L-functions

7.1. Construction

Let $\pi = \pi_\infty \otimes \pi_f$ be a cohomological automorphic representation of $G(\mathbb{A})$ of type $(\mathbf{k}, r)$ where $(\mathbf{k}, r) \in \mathbb{Z}^{\Sigma_F} \times \mathbb{Z}$ such that $k_\sigma \geqslant 2$, $k_\sigma \equiv r \bmod 2$ and $| \, r \, | \leqslant k_\sigma - 2$ for each $\sigma \in \Sigma_F$. Let $\mathfrak{c}$ be the conductor of π and suppose that $K_1(\mathfrak{c}p)$ satisfy the condition 2.1. Let k_π be a number field containing the normal closure of F and the field of definition of π_f. Let L be a p-adic field containing k_π. We call λ the dominant character attached to the data $(\mathbf{k}, r)$ as in 2.4. Let $\mathbf{f}_\pi \in \mathbf{S}_{(\mathbf{k},r)}(K_1(\mathfrak{c}))$ be the newform attached to π (here $\mathbf{S}_{(\mathbf{k},r)}(K_1(\mathfrak{c}))$ is the space $\mathbf{S}_{(\mathbf{k},\mathbf{w})}(K_1(\mathfrak{c}),\mathbb{C})$ with $\mathbf{w} = (\frac{k_\sigma - r}{2})_{\sigma\in\Sigma_F}$ in the notations of [10]). We suppose that the following condition is satisfied:

Hypothesis 7.1. — *There exists a p-stabilisation of $\mathbf{f}_\pi$, denoted $\mathbf{f} \in \mathbf{S}_{(\mathbf{k},r)}(K_1(\mathfrak{c}) \cap K_0(p)) \subset \mathbf{S}_{(\mathbf{k},r)}(K_1(\mathfrak{c}p))$, such that if we denote by $a_p \in \overline{\mathbb{Q}}$ the eigenvalue of $\mathbf{f}$ with respect to the Hecke operator U_p, then we have:*

$$v_p\left(\mathbf{inc}_p\left(p^{\sum_{\sigma\in\Sigma_F}\frac{k_\sigma-2-r}{2}} a_p\right)\right) < k^0 - 1,$$

where $k^0 = \min\{k_\sigma \mid \sigma \in \Sigma_F\}$.

Using a result of Matsushima–Shimura–Harder ([10, Proposition 3.1]) we obtain a class $\delta_{\mathbf{1}}(f) \in H^d_{\mathrm{cusp}}(Y_1(\mathfrak{c}p), \mathcal{L}(\mathbb{V}^\vee_\lambda(\mathbb{C})))$. Let $H^d_{\mathrm{cusp}}(Y_1(\mathfrak{c}p), \mathcal{L}(\mathbb{V}^\vee_\lambda(\mathbb{C})))[\mathbf{f},\mathbf{1}]$ be the space of the Hecke eigenclasses in $H^d_{\mathrm{cusp}}(Y_1(\mathfrak{c}p), \mathcal{L}(\mathbb{V}^\vee_\lambda(\mathbb{C})))$ with the same eigenvalues that $\mathbf{f}$ and sign $(1,\dots,1) \in \mathbb{Z}^{\Sigma_F}$. This space is 1-dimensional (see [10, §8]), then fixing a period $\Omega_\pi \in \mathbb{C}^\times$ we obtain a well determined class:

$$\phi_{\mathbf{f}} \in H^d_c(Y_1(\mathfrak{c}p), \mathcal{L}(\mathbb{V}^\vee_\lambda(L))).$$

By construction we have $U^0_p\phi_{\mathbf{f}} = \alpha\phi_{\mathbf{f}}$ with $\alpha = \mathbf{inc}_p(p^{\sum_{\sigma\in\Sigma_F}\frac{k_\sigma-2-r}{2}} a_p)$. By hypothesis $v_p(\alpha) < k^0 - 1$ and then by Theorem 5.1 there exists an unique

$$\Phi_{\mathbf{f}} \in H^d_c(Y_1(\mathfrak{c}p), \mathcal{L}(\mathcal{D}_\lambda(L))),$$

whose specialization is $\phi_{\mathbf{f}}$ and $U_p\Phi_{\mathbf{f}} = \alpha\Phi_{\mathbf{f}}$. Using the construction described in 6.3 we define:

$$\mu_{\mathbf{f}} := \alpha^{-1}\mu_{\Phi_{\mathbf{f}}} \in \mathcal{D}(\mathrm{Gal}_p, L)$$

Theorem 7.2. — *The distribution $\mu_{\mathbf{f}} \in \mathcal{D}(\mathrm{Gal}_p, L)$ is h-admissible where $h = v_p(\alpha)$. Let $\chi : \mathrm{Gal}_p \to L^\times$ be a finite order character of F such that $\chi_\sigma(-1) = 1$ for each $\sigma \in \Sigma_F$, then we have:*

$$\mu_{\mathbf{f}}(\chi) = \mathbf{inc}_p\left(\frac{L^p(\pi\otimes\chi, 1)\tau(\chi)}{\Omega_\pi}\right)\prod_{\mathfrak{p}|p} Z_{\mathfrak{p}},$$

here $L^p(\pi\otimes\chi, s)$ is the L-function of π twisted by χ without the Euler factor in p, $\tau(\chi)$ is the Gauss sum as defined in [9, §2.5], *and:*

$$Z_{\mathfrak{p}} = \begin{cases} \alpha_{\mathfrak{p}}^{-\mathrm{cond}(\chi_{\mathfrak{p}})} & \text{if } \chi_{\mathfrak{p}} \text{ is ramified} \\ \frac{\chi_{\mathfrak{p}}(\varpi_{\mathfrak{p}})^{-d_{\mathfrak{p}}}(1-\alpha_{\mathfrak{p}}^{-1}\chi_{\mathfrak{p}}(\varpi_{\mathfrak{p}})^{-1}N_{F/\mathbb{Q}}(\mathfrak{p})^{-1})}{1-\alpha_{\mathfrak{p}}\chi_{\mathfrak{p}}(\varpi_{\mathfrak{p}})} & \text{if not.} \end{cases}$$

where, $d_{\mathfrak{p}}$ is the $\mathfrak{p}$-adic valuation of the different of F, $\alpha_{\mathfrak{p}} = \mathbf{inc}_p(a_{\mathfrak{p}})$ and $a_{\mathfrak{p}}$ is the eigenvalue of $\mathbf{f}$ with respect to the hecke operator $U_{\mathfrak{p}}$.

In this statement we use Remark 4.6 to see $\chi : \mathrm{Gal}_p \to L^\times$ as a finite order Hecke character of F. Remark that from [10, Theorem 8.1] we know that $\frac{L(\pi\otimes|\cdot|^n_{\mathbb{A}_F}\otimes\chi,1)}{\Omega_\pi} \in \overline{\mathbb{Q}}$ where n is a integer such that $\frac{-(k^0-2)+r}{2} \leqslant n \leqslant \frac{(k^0-2)+r}{2}$ and χ is any finite order Hecke character of F. We prove this theorem in the next two subsections. In the next subsection we recall the evaluations described in [9] and we make explicit the relation with our evaluations. This explicit relation allow us relate our construction with L-values. We would like to remark that a basic problem in our construction is the lack of uniqueness of our p-adic L-functions. This problem is a consequence of the fact that Theorem 1.2 does not guarantee the interpolation of *enough* critical values. This problem is settled in the ongoing work [3]. We refer to [12] for a precise study of distributions on Galois groups and the problem of uniqueness.

7.2. A computation of Dimitrov

We follow [9] to define evaluations on $H^d_c(Y_{K_1(\mathfrak{c}p)}, \mathcal{L}(\mathbb{V}^\vee_\lambda(L)))$, moreover we relate these evaluations with some critical values of $L(\pi, s)$. Here we will denote $\mathbb{V}^\vee_\lambda(L)$ by $\mathbb{V}^\vee_\lambda$ and $\mathcal{D}_\lambda(L)$ by $\mathcal{D}_\lambda$. These evaluations are defined in the same way as in 6.2:

- Using the automorphic cycle $C_{K_1(\mathfrak{c}p),n}$ we obtain:

$$H^d_c(Y_{K_1(\mathfrak{c}p)}, \mathcal{L}(\mathbb{V}^\vee_\lambda)) \longrightarrow H^d(X_n, C^\times_{K_1(\mathfrak{c}p),n}\mathcal{L}(\mathbb{V}^\vee_\lambda));$$

- Let $\mathcal{L}_n(\mathbb{V}^\vee_\lambda)$ be the sheaf on X_n obtained by considering the action $u \in U(p^n)$ on $\mathbb{V}^\vee_\lambda$ by the matrix $\left(\begin{smallmatrix} u & 0 \\ 0 & 1 \end{smallmatrix}\right)$ on the right. Then the right action of $\left(\begin{smallmatrix} 1 & -1 \\ 0 & p^n \end{smallmatrix}\right)$ on $\mathbb{V}^\vee_\lambda$ gives $C^\times_{K_1(\mathfrak{c}p),n}\mathcal{L}(\mathbb{V}^\vee_\lambda) \to \mathcal{L}_n(\mathbb{V}^\vee_\lambda)$ a morphism of sheaves over X_n. Then we obtain: $H^d(X_n, C^\times_{K_1(\mathfrak{c}p),n}\mathcal{L}(\mathbb{V}^\vee_\lambda)) \to H^d(X_n, \mathcal{L}_n(\mathbb{V}^\vee_\lambda))$;
- Consider the morphism $\mathbb{V}^\vee_\lambda \to L$ defined by $\varphi \mapsto \binom{\mathbf{k}-2\mathbf{t}}{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}}\varphi(f_\star)$, where $f_\star \in \mathbb{V}^\vee_\lambda$ is uniquely determined by the condition $f_\star\left(\begin{smallmatrix} 1 & z \\ 0 & 1 \end{smallmatrix}\right) = z^{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}}$ for all $z \in F \otimes_{\mathbb{Q}} L$. Then we obtain $H^d(X_n, \mathcal{L}_n(\mathbb{V}^\vee_\lambda)) \to H^d(X_n, L) \simeq L^{\mathrm{Cl}^+_F(p^n)}$;
- Putting all these steps together we get:

$$\mathbf{ev}^{\mathrm{Cl}}_{K_1(\mathfrak{c}p),n} : H^d_c(Y_{K_1(\mathfrak{c}p)}, \mathcal{L}(\mathbb{V}^\vee_\lambda)) \longrightarrow L^{\mathrm{Cl}^+_F(p^n)}.$$

Lemma 7.3. — *We put* $\mathrm{cr}:\mathcal{D}_\lambda^+\to L$, $\mu\to \mathrm{cr}(\mu)=\binom{\mathbf{k}-2\mathbf{t}}{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}}\mu(z^{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}})$. *Then we have the following commutative diagram:*

$$\begin{array}{ccc}
\mathbf{ev}_{K_1(\mathfrak{c}p),n}: H^d_c(Y_1(\mathfrak{c}p),\mathcal{L}(\mathcal{D}_\lambda)) & \longrightarrow & (\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^n)} \\
\downarrow & & \downarrow{\scriptstyle \mathrm{cr}} \\
p^{n\frac{\mathbf{k}-2\mathbf{t}-r\mathbf{t}}{2}}\mathbf{ev}^{\mathrm{Cl}}_{K_1(\mathfrak{c}p),n}: H^d_c(Y_1(\mathfrak{c}p),\mathcal{L}(\mathbb{V}_\lambda^\vee)) & \longrightarrow & L^{\mathrm{Cl}_F^+(p^n)}
\end{array}$$

Proof. — If we denote by $\pi:\mathcal{D}_\lambda\to\mathbb{V}_\lambda^\vee$ the projection, it is I-equivariant, moreover for each $\mu\in\mathcal{D}_\lambda$ we have:

$$\pi\left(\mu*\begin{pmatrix}1&-1\\0&p^n\end{pmatrix}\right)=p^{n\frac{\mathbf{k}-2\mathbf{t}-r\mathbf{t}}{2}}\pi(\mu)\cdot\begin{pmatrix}1&-1\\0&p^n\end{pmatrix}.$$

So comparing the definitions of $\mathbf{ev}^{\mathrm{Cl}}_{K_1(\mathfrak{c}p),n}$ and $\mathbf{ev}_{K_1(\mathfrak{c}p),n}$ the lemma follows. □

Lemma 7.4. — *Let $\phi_{\mathbf{f}}$ be as in 7.1, and for each $n\geqslant 1$ we denote $\mathbf{ev}^{\mathrm{Cl}}_{K_1(\mathfrak{c}p),n}(\phi_{\mathbf{f}})=(a_{\mathbf{y},\mathbf{f}})_{\mathbf{y}\in\mathrm{Cl}_F^+(p^n)}\in L^{\mathrm{Cl}_F^+(p^n)}$. Let $\chi:\mathrm{Gal}_p\to L^\times$ be a finite order character such that $\chi_\sigma(-1)=1$ for each $\sigma\in\Sigma_F$. Let $n>0$ large enough such that χ factorizes through the projection $\mathrm{Gal}_p\to\mathrm{Cl}_F^+(p^n)$, then we have:*

$$p^{n\frac{\mathbf{k}-2\mathbf{t}-r\mathbf{t}}{2}}\alpha^{-n}\sum_{\mathbf{y}\in\mathrm{Cl}_F^+(p^n)}\chi(\mathbf{y})a_{\mathbf{y},\pi}=\mathbf{inc}_p\left(\frac{L^p(\pi\otimes\chi,1)\tau(\chi)}{\Omega_\pi}\right)\prod_{\mathfrak{p}|p}Z_{\mathfrak{p}}.$$

Proof. — Using the notations of [9, §1.5] with $w=(k_\sigma-2)_{\sigma\in\Sigma_F}$ and $w_0=r$, and the commutative diagram (2.3) we obtain:

$$\begin{aligned}S^{w,w_0}_{K_1(\mathfrak{c}p),1,n}(\phi_{\mathbf{f}})&=p^{n\frac{\mathbf{k}-2\mathbf{t}-r\mathbf{t}}{2}}\alpha^{-n}\mathbf{ev}^{\mathrm{Cl}}_{K_1(\mathfrak{c}p),n}(\phi_{\mathbf{f}})\\&=p^{n\frac{\mathbf{k}-2\mathbf{t}-r\mathbf{t}}{2}}\alpha^{-n}(a_{\mathbf{y},\mathbf{f}})_{\mathbf{y}\in\mathrm{Cl}_F^+(p^n)}.\end{aligned}$$

Finally the lemma result from the calculations given in the proof of [9, Theorem 2.4]. □

7.3. Proof of Theorem 7.2

We have that $U_p(\Phi_{\mathbf{f}})=\alpha\Phi_{\mathbf{f}}$, then from Lemma 6.10 we deduce that $\mu_{\mathbf{f}}$ is an h-admissible distribution where $h=v_p(\alpha)$.

Let $\chi:\mathrm{Gal}_p\to L^\times$ be a finite order character such that $\chi_\sigma(-1)=1$ for each $\sigma\in\Sigma_F$. Let $n\geqslant 1$ be such that the conductor of χ is divided by $p^n\mathcal{O}_F$. Then we can consider χ to be defined on $\mathrm{Cl}_F^+(p^n)$.

We write $\mathrm{ev}_{K_1(\mathfrak{c}p),n}(\Phi_{\mathbf{f}}) = (\nu_{\mathbf{y}})_{\mathbf{y}\in \mathrm{Cl}_F^+(p^n)} \in (\mathcal{D}_\lambda^+)^{\mathrm{Cl}_F^+(p^n)}$. From definition in 4.3.3 and Lemma 6.8, we obtain that if $\mathbf{x} \in \mathrm{Cl}_F^+$ and $\mathbf{y} \in \mathrm{Cl}_F^+(p^n)_{\mathbf{x}}$ then we have:

$$\nu_{\mathbf{y}}^\times(\chi_{\mathbf{x}}) = \binom{\mathbf{k}-2\mathbf{t}}{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}}\chi(\mathbf{y})\nu_{\mathbf{y}}(z^{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}}),$$

here $\chi_{\mathbf{x}}$ is given by the composition of the homeomorphism $r_{\mathbf{x}} : \mathrm{Gal}_p^\circ \to \mathrm{Gal}_{p,\mathbf{x}}$ and χ.

As before we write $\mathbf{ev}_{K_1(\mathfrak{c}p),n}^{\mathrm{Cl}}(\phi_{\mathbf{f}}) = (a_{\mathbf{y},\mathbf{f}})_{\mathbf{y}\in\mathrm{Cl}_F^+(p^n)}$. Then we have:

$$\begin{aligned}
\mu_{\mathbf{f}}(\chi) &= \alpha^{-1}\mu_{\Phi_{\mathbf{f}}}(\chi)\\
&= \alpha^{-n}\sum_{\mathbf{x}\in\mathrm{Cl}_F^+}\sum_{\mathbf{y}\in\mathrm{Cl}_F^+(p^n)_{\mathbf{x}}}\nu_{\mathbf{y}}^\times(\chi_{\mathbf{x}})\\
&= \alpha^{-n}\sum_{\mathbf{x}\in\mathrm{Cl}_F^+}\sum_{\mathbf{y}\in\mathrm{Cl}_F^+(p^n)_{\mathbf{x}}}\binom{\mathbf{k}-2\mathbf{t}}{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}}\chi(\mathbf{y})\nu_{\mathbf{y}}(z^{\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}})\\
&= p^{n\frac{\mathbf{k}-2\mathbf{t}+r\mathbf{t}}{2}}\alpha^{-n}\sum_{\mathbf{y}\in\mathrm{Cl}_F^+(p^n)}\chi(\mathbf{y})a_{\mathbf{y},\pi},\\
&= \mathbf{inc}_p\left(\frac{L^p(\pi\otimes\chi,1)\tau(\chi)}{\Omega_\pi}\right)\prod_{\mathfrak{p}|p}Z_{\mathfrak{p}}.
\end{aligned}$$

The second equality follows from Lemma 6.1, the fourth from Lemma 7.3 and the last one from Lemma 7.4.

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