ANNALES

DE

L'INSTITUT FOURIER

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Tome 68, nº 3 (2018), p. 1121-1123.

<http://aif.cedram.org/item?id=AIF_2018__68_3_1121_0>

Ann. Inst. Fourier, Grenoble
68, 3 (2018) 1121-1123

CORRIGENDUM TO "NON-REDUCTIVE AUTOMORPHISM GROUPS, THE LOEWY FILTRATION AND K-STABILITY"

Annales de l'institut Fourier, vol. 66 (2016), n°5, 1895–1921

by Giulio CODOGNI & Ruadhaí DERVAN

There is an error in the proof of [2, Lemma 2.4], where the multiplicativity of the Loewy filtration is claimed (pointed out to us by Y. Odaka). We are not aware of any examples in which the Loewy filtration of a polarised variety is not multiplicative, and indeed in all examples given it is multiplicative. Moreover, multiplicativity is always true in the toric case; we briefly sketch the argument below.

The error is as follows. To start with, the results of [1] used do not apply in our situation. This in itself is not a serious issue, as the Jacobson ideal can be replaced by the ideal generated by the image of the Lie algebra of unipotent radical U of $\mathrm{Aut}(X, L)$ inside the universal enveloping algebra of $\mathfrak{gl}(H^0(X, kL))$. The main problem is that the containment

$$J(A)^i R_\ell \cdot J(A)^j R_s \subset J(A)^{i+j} R_{\ell+s}$$

does not hold in the general. Indeed, it is not true that

$$(g^* s)(h^* t) = (gh)^* st \tag{0.1}$$

where g and h are in U, and s and t are sections.

The proof of multiplicativity in the toric case is as follows (which arose from discussions with H. Süss). Let $T \subset \mathrm{Aut}(X, L)$ be a maximal torus. As the Loewy filtration is equivariant with respect to the action of $\mathrm{Aut}(X, L)$, it is enough to prove the claim when s and t are eigenvectors of T, g and h are in U, and $g = \exp(A)$ and $h = \exp(B)$, with A and B eigenvectors of

Keywords: K-stability, reductive groups, Kähler–Einstein metrics, radical filtration.

T. In this case, both sides of equation (0.1) are eigenvectors for the action of T with the same eigenvalue. A key feature of toric varieties is that the eigenspaces are one dimensional, so equation (0.1) is true up to a scalar, and this is enough to prove the multiplicativity of the Loewy filtration. We plan to return to the Loewy filtration in the toric setting in future work.

While we are not aware of any examples in which the Loewy filtration of a polarised variety is not multiplicative, there is a related example provided to us by Y. Odaka in which multiplicativity fails. Consider $\mathbb{P}^3$ with homogeneous coordinates x_0, x_1, x_2, x_3 with an action of $a \in \mathbb{C}_+$ by $x_0 \to x_0, x_1 \to x_1, x_2 \to x_2+ax_0, x_3 \to x_3+ax_1$. It is then a straightforward calculation to show that the induced Loewy filtration is not multiplicative. Remark that the Loewy filtration induced from $\mathrm{Aut}(\mathbb{P}^3, \mathcal{O}(1))$ is trivial since its automorphism group is reductive.

In general the Loewy filtration still produces a sequence of test configurations; it may be that Conjecture B should be modified to state that the Loewy filtration destabilises in a suitable sense of uniform K-polystability.

There is also another smaller error in [2, Section 1.2]. First, we did not remark that the polynomial Donaldson–Futaki invariant introduced in Definition 1.4 actually corresponds to the Chow_∞-weight used for example in [3]. As explained in [3], this invariant is equal to the Donaldson–Futaki invariant when the filtration is finitely generated, but it is not clear if there is any inequality in the general case. An example in which the two invariants differ is [3, Example 4].

In particular, the proof of [2, Lemma 1.14] is incorrect. The mistake in the proof is that the weight function is equal to the weight polynomial $w(k)$ just for sufficiently large k, and this threshold can actually grow with the r appearing in the claim of the Lemma. This also invalidates [2, Theorem 1.10], and item (ii) of [2, Corollary 1.14] is most likely not equivalent to (i) and (iii).

This does not affect any of the examples, as the filtration is finitely generated in each case. Finite generation follows as the filtration is by vanishing order in each example (along the fixed locus of the unipotent radical of the automorphism group), and the argument of Proposition 3.1 implies vanishing order filtrations are finitely generated on toric varieties.

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Manuscrit reçu le 25 janvier 2018,
révisé le 5 février 2018,
accepté le 5 février 2018.

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