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REDUCIBLE REPRESENTATIONS OF ABELIAN GROUPS

by Aharon ATZMON

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1. Introduction.

Irreducible representations of the groups $\mathbb{Z}$ and $\mathbb{R}$ in a nuclear Fréchet space are constructed in [3] and [4], respectively. As observed in [4], the result of C. Read [23] implies that there exists an irreducible representation of $\mathbb{Z}$ in the Banach space ℓ_1. On the other hand, it is not known whether every representation of an abelian group in an infinite dimensional complex Hilbert space, or reflexive Banach space, is reducible. For the group $\mathbb{Z}$, this problem is equivalent to the problem of whether every (bounded linear) invertible operator on such a space has a nontrivial bi-invariant subspace (i.e., a common nontrivial invariant subspace with its inverse).

Some sufficient conditions for the reducibility of a representation of an abelian group in a Banach space are given in [3] and [9]. They involve certain assumptions on the growth and spectrum of the operators in the range of the representation.

In this paper we prove the reducibility of representations of an abelian group in a reflexive Banach space, under assumption of different type. As an application of our main result we obtain a positive solution to the translation invariant subspace problem for weighted L^p spaces on locally

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compact abelian groups, for even weights and $1 < p < \infty$. For the group $\mathbb{Z}$ this was proved in [8], but the proof there does not carry over to the general case.

In Section 2 we state our main results, and in Section 3 we give their proofs. Section 4 is devoted to comments, problems and further results.

2. Main results.

In what follows, G denotes an abelian group, X an infinite dimensional complex Banach space, and T a representation of G in X, that is, a homomorphism of G into the group of bounded linear invertible operators on X. We do not assume in general that G is a topological group. We recall that the representation T is called *reducible* if the operators $T(g), g \in G$, have a common nontrivial (closed) invariant subspace.

A conjugate linear mapping J of X into itself will be called an *involution*, if J^2 is the identity operator on X.

Our main result is

THEOREM 1. — *Assume that X is reflexive, and that there exists a continuous involution J on X such that*

$$JT(g) = T(-g)J, \quad \forall g \in G.$$

If there exist nonzero elements u in X and v in the dual space X^*, such that $Ju = u$ and the function on G defined by $g \longrightarrow \langle T(g)u, v\rangle$ is positive definite, then T is reducible.

Before describing applications of the theorem, we introduce some notations and definitions. If f is a complex function on G, and t is in G, we shall denote by f_t the t-translate of f, that is, the function on G defined by $g \longrightarrow f(g-t)$; the function $g \longrightarrow \overline{f(-g)}$ will be denoted by f^*.

If G is a locally compact topological group with Haar measure dg, and E is an infinite dimensional complex Banach space of locally integrable functions on G (with respect to dg), we shall say that E is *admissible*, if for every compact subset K of G, there exists a positive constant $c(K)$, such that

$$(*) \qquad \int_K |f(g)|dg \leqslant c(K)\|f\|_E\,, \quad \forall f \in E.$$

Remark. — We identify functions in E whose difference vanishes on a set of zero Haar measure.

We shall say that the Banach space E is *translation invariant* if for every $f \in E$ and $t \in G$, the function f_t is in E, and that E is *Hermitian*, if for every $f \in E$, the function f^* is in E.

As we shall prove in Section 2, the following is a consequence of Theorem 1.

THEOREM 2. — *Assume that G is a locally compact group, and that E is an admissible translation invariant Banach space of functions on G, which is Hermitian and reflexive. If E contains a nonzero bounded Borel function u with compact support such that $u = u^*$, then it has a nontrivial translation invariant subspace.*

Remarks. — For the group $G = \mathbb{Z}$, the theorem was proved in [8] under stronger assumptions. The proof there does not carry over the general case, even for discrete groups.

¿From Theorem 2 we obtain an affirmative answer to the translation invariant subspace problem for weighted L^p spaces on locally compact abelian groups, for even weights and $1 < p < \infty$. To describe this problem, we need some additional definitions.

Assume that G is a locally compact group. A positive Borel function ω on G will be called a *weight*, if for every compact subset K of G

$$0 < \inf_{g \in K} \omega(g) \leqslant \sup_{g \in K} \omega(g) < \infty,$$

and for every t in G

$$\sup_{g \in G} \frac{\omega(g+t)}{\omega(g)} < \infty.$$

For a weight ω and $1 \leqslant p < \infty$, we shall denote by $L^p_\omega(G)$ the Banach space of all complex functions f on G, such that $f\omega \in L^p(G)$, equipped with the induced norm

$$\|f\|_{L^p_\omega(G)} = \|f\omega\|_{L^p(G)}.$$

It is easily verified that $L^p_\omega(G)$ is an admissible translation invariant Banach space of functions on G, which contains all continuous functions with compact support.

The translation invariant subspace problem for weighted L^p spaces on abelian groups asks, whether for every weight ω on a locally compact non-compact abelian group G, the space $L^p_\omega(G)$ has a nontrivial translation invariant subspace. An affirmative answer was given by Domar [13] for $G = \mathbb{R}$. His proof uses boundary values of certain analytic functions in the upper half plane, and does not carry over to general groups, in particular to the integer group $\mathbb{Z}$. For this group the problem is open for every $1 \leqslant p < \infty$; for $p = 2$, it is equivalent to the hyperinvariant subspace problem for bilateral weighted shifts on Hilbert space (see [24], Question 23 and [20], Sections 3 and 4).

A rather extensive literature appeared on the translation invariant subspace problem for weighted L^p spaces on abelian groups, and related aspects of that problem, especially for the group $\mathbb{Z}$ and $p = 2$, and affirmative solutions were obtained for weights that satisfy certain regularity and growth conditions (cf. [1–8], [12–20], [10], [24]). Some comments on the problem for the group $\mathbb{R}$ (the discrete real line) appear in [14].

It is easy to see that the space $L^p_\omega(G)$ is Hermitian, if and only if, $\sup_{g\in G} \frac{\omega(-g)}{\omega(g)} < \infty$. If this holds, we shall say that the weight ω is *essentially even.*

The following is an immediate consequence of Theorem 2.

THEOREM 3. — *If ω is an essentially even weight on the locally compact group G, and $1 < p < \infty$, then $L^p_\omega(G)$ has a nontrivial translation invariant subspace.*

3. Proofs.

For the proof of Theorem 1, we need the following

LEMMA. — *Assume that φ is a positive definite function on G, let $t \in G$, and $s \in [-1,1]$. Then the functions $f = \varphi - \frac{s}{2}(\varphi_t + \varphi_{-t})$ and $h = \varphi - \frac{s}{2i}(\varphi_t - \varphi_{-t})$ are also positive definite.*

Proof. — Let V denote the complex vector space of all complex functions on G with finite support. For every complex function ψ on G, let L_ψ denote the sesquilinear form on V defined by

$$L_\psi(a,b) = \sum \psi(p-q)a(p)\overline{b(q)}, \quad a,b \in V,$$

the sum being extended over all p,q in G. Recall that the function ψ is called positive definite, if the form L_ψ is positive, that is $L_\psi(a,a) \geqslant 0$ for every a in V.

To show that the functions f and h are positive definite, fix a in V. Using the fact that a sesquilinear positive form on a complex vector space is Hermitian, we get by a simple computation that

$$L_f(a,a) = L_\varphi(a,a) - s\mathrm{Re}L_\varphi(a,a_t)$$

and

$$L_h(a,a) = L_\varphi(a,a) - sImL_\varphi(a,a_t).$$

Thus noticing that $L_\varphi(a_t,a_t) = L_\varphi(a,a)$, and applying the Schwarz inequality for positive Hermitian forms, we obtain that

$$L_f(a,a) \geqslant (1-|s|)L_\varphi(a,a) \geqslant 0$$

and

$$L_h(a,a) \geqslant (1-|s|)L_\varphi(a,a) \geqslant 0,$$

which proves the assertion. □

Proof of Theorem 1. — Consider the set $X_0 = \{x \in X : Jx = x\}$. It is a closed real vector subspace of X, hence a real Banach space with respect to the norm of X. It follows from the properties of J that the mapping $x \to \frac{1}{2}(x + Jx)$ is a bounded projection from X (regarded as a real Banach space) on X_0, and $X = X_0 + iX_0$. Since X is reflexive, it follows from this decomposition that X_0 is also reflexive, and therefore by a result of Lindenstrauss [22] (see also [11]), it has an equivalent norm which is smooth (i.e., is Gateâux differentiable in the complement of the zero vector). Thus we may, and we shall, assume that the norm of X_0 is smooth.

For every g in G, consider the operators

$$A(g) = \frac{1}{2}(T(g) + T(-g)), \quad \text{and} \quad B(g) = \frac{1}{2i}(T(g) - T(-g)).$$

The assumption about the relation between T and J, implies that these operators commute with J, and therefore they map the space X_0 into itself. It follows from these facts, that if there exists a nontrivial subspace M of X_0 which is invariant under the operators $A(g)$ and $B(g)$ for all $g \in G$, then $M + iM$ is a nontrivial subspace of X which is invariant under all the operators $T(g) = A(g) + iB(g)$, $g \in G$, so the assertion of the theorem holds.

To establish the existence of such a subspace M, consider the set C of all elements x in X_0, such that the function on G defined by $g \to \langle T(g)x, v\rangle$, is positive definite. It is clear that C is a convex subset of X_0, and since the pointwise limit of a sequence of positive definite functions is also positive definite, it is closed. It is a proper subset of X_0, since if x is an element in X_0 such that $\langle x, v\rangle \neq 0$, then either x or $-x$ is not in C, since the value of a positive definite function at the zero element of the group is nonnegative.

It is an elementary fact that if a closed subset of a Banach space of dimension greater than one contains a nonzero element, then its boundary also contains a nonzero element. Thus, since C contains the nonzero vector u, its boundary also contains a nonzero vector. Therefore, since the distance of a vector in a reflexive Banach space to a closed convex subset is attained (cf. [21], p. 340), it follows that there exists an element z of X_0 that is not in C, and a nonzero element y in C, such that the distance from z to C is $\|z-y\|$. Let M denote the closed linear span in X_0 of the vectors $A(g)y$, $B(g)y$, $g \in G$. We claim that this subspace has the desired properties.

First, M is invariant under the operators $A(g)$ and $B(g)$ for all g in G, since their linear span coincides with the algebra they generate. M is not the zero space, since it contains the vector y. In order to prove that $M \neq X_0$, we have to show that there exists a nonzero element of X_0^*, which annihilates all the vectors $A(g)y, B(g)y, g \in G$. Since the norm of X_0 is smooth and the vector $\omega = z - y$ is not zero, there exists a unique unit vector q in X_0^* such that $\langle \omega, q\rangle = \|\omega\|$ (see [21], Section 20). We claim that q annihilates all the vectors above. To see this, fix t in G, and consider the functions F_1 and F_2 on $[-1,1]$, defined by $s \to \|\omega + sA(t)y\|$ and $s \to \|\omega + sB(t)y\|$, respectively. By the lemma, the vectors $y - sA(t)y$ and $y - sB(t)y$ are in C for every $s \in [-1,1]$, and therefore, since the distance of z from C is $\|\omega\|$, the functions F_1 and F_2 have a minimum at $s = 0$. On the other hand, since the norm X_0 is smooth, these functions are differentiable at $s = 0$, and the derivatives are $\langle A(t)y, q\rangle$ and $\langle B(t)y, q\rangle$, respectively. (See [21], p. 349). So q annihilates all the vectors $A(g)y$, $B(g)y$, $g \in G$, and the theorem is proved. □

Proof of Theorem 2. — For every t in G, let S_t denote the linear transformation on E that sends a function f in E to the function f_t. Since E is translation invariant these transformations map E into itself, and condition $(*)$ and the closed graph theorem imply that they are bounded. Thus the mapping $t \to S_t$ is a representation of G in E. Since E is Hermitian, the mapping J on E defined by $Jf = f^*, f \in E$, is an involution on that space, and another application of the closed graph theorem shows that it is continuous, and it is readily verified that

$$JS_t = S_{-t}J, \quad \forall t \in E.$$

Let u be a nonzero bounded Borel function in E with compact support such that $u = u^*$ (which exists by the assumption on E) and let v be the linear functional on E defined by

$$\langle f, v\rangle = \int_G f(g)\overline{u(g)}dg, \quad f \in E.$$

It follows from condition $(*)$ that v is in E^*, and it is not the zero vector since $\langle u, v\rangle \neq 0$. A standard computation which uses the translation invariance of Haar measure, shows that the function on G defined by $g \to \langle S_g u, v\rangle$ is positive definite, and therefore the conclusions of the theorem follow from Theorem 1. □

4. Comments, problems and further results.

It is worth noting that the reducibility of the representations in Theorem 1 is not achieved in general by spectral decomposition of the operators in their range or their adjoints. For example, if ω is the even weight on $\mathbb{Z}$ defined by

$$\omega(n) = \exp\left\{\frac{|n|}{\log(|n|+e)}\sin[\log\log\log(|n|+e^3)]\right\}, \quad n \in \mathbb{Z},$$

and T is the representation of $\mathbb{Z}$ in E defined by $T(n) = S^n$, $n \in \mathbb{Z}$, where S is the operator on $L^2_\omega(\mathbb{Z})$ that sends the sequence $\{a(k)\}$ to the sequence $\{a(k-1)\}$, then it follows from the results in [6], that the spectrum of the restriction of each of the operators $T(n)$, $n \in \mathbb{Z}$ to a common nonzero invariant subspace is the unit circle, and the same is true for the adjoints of these operators.

If G is a locally compact group, and E is a translation invariant admissible Banach space of functions on G that satisfies the conditions of Theorem 2, then it follows from its proof, that there exists a nonzero function φ in the set C introduced in the proof of Theorem 1, which generates a nontrivial translation invariant subspace of E (i.e., the closed linear span in E of the translates $\varphi_t, t \in G$, is different from E). If the group G is discrete, we may choose the function u in the proof of Theorem 2 to be the characteristic function of the set $\{0\}$, and in this case the corresponding functional v is evaluation at 0. For this functional, the set C consists of all positive definite functions on G that are in E. Thus we get that if G is discrete, then there exists a positive definite function which generates a nontrivial translation invariant subspace of E.

Another result which follows from Theorem 1 is

THEOREM 4. — *Let G be a locally compact abelian group with dual group Γ. Assume that E is a reflexive admissible Banach space of functions on G, which is invariant under multiplication by elements of Γ. If E is self-adjoint (that is, for every function f in E, the function $\overline{f}$ is also in E), and contains a nonzero real valued bounded Borel function with compact support, then it has a nontrivial subspace which is invariant under multiplication by elements of Γ.*

Proof. — For every γ in Γ, let R_γ denote the operator of multiplication by γ on E, and let J denote the involution of complex conjugation on E. Then $JR_\gamma = R_{-\gamma}J$, $\forall\gamma \in \Gamma$, and an application of the closed graph theorem shows that the operators R_γ and J are continuous. Hence the mapping $\gamma \to R_\gamma$ is a representation of Γ in E. Let u be a nonzero real valued bounded Borel function with compact support in E, and let v be the linear functional on E defined by

$$\langle f, v\rangle = \int_G f(g)u(g)dg, \quad f \in E.$$

As in the proof of Theorem 2, we see that v is a nonzero element of E^*. It is easily verified that the function on Γ defined by $\gamma \to \langle R_\gamma u, v\rangle$ is positive definite, and the conclusion follows from Theorem 1. □

In what follows $\mathbb{T}$ denotes the circle group $\mathbb{R}/2\pi\mathbb{Z}$. The following is an immediate consequence of Theorem 4.

THEOREM 5. — *Assume that E is a Hilbert space of complex functions on $\mathbb{T}$ which is included in $L^1(\mathbb{T})$, and the embedding is continuous. If E is self-adjoint, contains a nonzero real valued function in $L^\infty(\mathbb{T})$, and is invariant under multiplication by the functions $e^{in\theta}, n \in \mathbb{Z}$, then it has a nontrivial subspace which is invariant under multiplication by these functions.*

We now give a concrete application of Theorem 5. Let ω be an even weight on $\mathbb{Z}$ which is increasing on $\mathbb{Z}_+$, and denote by E_ω the Hilbert space of all functions f in $L^2(\mathbb{T})$, for which the norm

$$\|f\| = \Big(\sum_{n\in\mathbb{Z}} |\hat{f}(n)|^2\omega(n)^2\Big)^{1/2}$$

is finite. (As usual we denote for a function f in $L^1(\mathbb{T})$ by $\hat{f}$ the sequence of its Fourier coefficients). It is easy to see that E_ω satisfies the conditions of Theorem 5, and therefore it has a nontrivial subspace which is invariant under multiplication by the functions $e^{in\theta}, n \in \mathbb{Z}$.

In addition, if we assume that the sequence ω satisfies the condition $\sum_{n=1}^{\infty} n^{-2} \log \omega(n) = \infty$, then by a theorem of Beurling (see [16], Theorem 4.2), there does not exist a nonzero function in E_ω that vanishes on a set of positive measure, so in this case, one cannot obtain the invariant subspace in Theorem 5, by considering the space of functions in E_ω that vanish a.e. on a set of positive measure, as in the case of for $E = L^2(\mathbb{T})$. If ω also satisfies the condition $\sum_{n=1}^{\infty} \frac{1}{\omega(n)^2} = \infty$, then one cannot take this invariant subspace to be the set of functions in E_ω that vanish at a fixed point in $\mathbb{T}$, since this condition on ω implies that the point evaluations are not continuous on E_ω. Thus if ω satisfies these two additional conditions, then the invariant subspaces of E_ω provided by Theorem 5 are not plainly visible.

The following natural question is of considerable interest.

Problem. — Does the conclusion of Theorem 5 hold without the assumption that E is self-adjoint?

As mentioned in Section 1, it is not known whether every invertible operator on a complex infinite dimensional reflexive Banach space has a nontrivial bi-invariant subspace, or equivalently, whether every representation of the group $\mathbb{Z}$ in such a space is reducible. Theorem 1 provides the following partial solution.

THEOREM 6. — *Assume that A is an invertible operator on a complex infinite dimensional reflexive Banach space X, and that there exists a continuous involution J on X such that $JA = A^{-1}J$. If there exist nonzero vectors u in X and v in X^* such that $Ju = u$, and the sequence $\{\langle A^n u, v\rangle\}_{n\in\mathbb{Z}}$ is positive definite, then A has a nontrivial bi-invariant subspace.*

Proof. — The assumptions imply that $JA^n = A^{-n}J$, $\forall n \in \mathbb{Z}$, and therefore the assertion follows by applying Theorem 1 to the representation $n \to A^n$ of $\mathbb{Z}$ in X. □

We conclude with a concrete application of this result. Let $1 \leqslant p < \infty$, and assume that $\lambda = \{\lambda(n)\}_{n\in\mathbb{Z}}$ is a sequence of nonzero real numbers, such that for every function f in $L^p(\mathbb{T})$, the sequences $\lambda\widehat{f}$ and $\frac{1}{\lambda}\widehat{f}$ are sequences of Fourier coefficients of functions in $L^p(\mathbb{T})$. Let Λ denote the linear transformation on $L^p(\mathbb{T})$, that sends a function f to the function whose sequence of Fourier coefficients is $\lambda\widehat{f}$. The assumptions on λ imply that Λ is an invertible operator on $L^p(\mathbb{T})$, and an application of the closed graph theorem shows that it is bounded. Let V be the operator on $L^p(\mathbb{T})$ of multiplication by the function $e^{i\theta}$, and denote by A the operator $V\Lambda$. It is invertible, and the problem whether for every sequence λ with the above properties, it has a nontrivial bi-invariant subspace, seems to be hard. For $p = 2$, it is equivalent to the translation invariant subspace problem for weighed ℓ^2 spaces on $\mathbb{Z}$.

We claim that if λ satisfies the additional condition $\lambda(-n)\lambda(n) = 1$, $\forall n \in \mathbb{Z}$, and $1 < p < \infty$, then A has a nontrivial bi-invariant subspace. Without loss of generality we may assume that $\lambda(0) = 1$. Let J denote the involution on $L^p(\mathbb{T})$ defined by $f \to e^{i\theta}\overline{f}$, denote by u the function $1 + e^{i\theta}$, and by v the functional $f \to \widehat{f}(0) + \widehat{f}(1)$ on $L^p(\mathbb{T})$. Simple computations show that $Ju = u$, $JA = A^{-1}J$, and

$$\langle A^n u, v\rangle = 2\delta_{n,0} + \delta_{n,1} + \delta_{n,-1}, \quad n \in \mathbb{Z}.$$

The last identity implies that the sequence $\{\langle A^n u, v\rangle\}_{n\in\mathbb{Z}}$ is positive definite, and the claim follows from Theorem 6.

For $p = 2$, the claim is equivalent to the fact that for an essentially even weight ω on $\mathbb{Z}$, the Hilbert space $\ell^2_\omega(\mathbb{Z})$ has a nontrivial translation invariant subspace.

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