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ON THE EMBEDDING OF 1-CONVEX MANIFOLDS WITH 1-DIMENSIONAL EXCEPTIONAL SET

by L. ALESSANDRINI & G. BASSANELLI *

Introduction.

Any 1-convex (i.e. strongly pseudoconvex) space X is a proper modification at finitely many points of a Stein space Y; X is called embeddable if it is biholomorphic to a closed analytic subspace of $\mathbb{C}^m \times \mathbb{CP}_n$, for suitable m and n. C. Banica [2] and Vo Van Tan [15] had shown that every 1-convex surface is embeddable; the case when $\dim X > 2$ has been studied in some papers of M. Coltoiu and Vo Van Tan (see [5] and [16] for the references), mainly when X is smooth and its exceptional set S is 1-dimensional. Up to now, the best result is the following:

THEOREM. — *Let X be a 1-convex manifold with 1-dimensional exceptional set S. Then X is embeddable, with a possible exception given by*

$\dim X = 3$ *and S contains a rational curve of type* $(-1,-1)$ *or* $(0,-2)$ *or* $(1,-3)$.

In these three exceptional cases, there are some examples of non-embeddable manifolds, only one of which is completely clarified (see [5]); anyway, the above result gives no criterion in order to check the embedding

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property in these situations. In this paper, we give a precise criterion, which turns out to be topological, namely:

THEOREM I. — *Let X be a 1-convex manifold with 1-dimensional exceptional set S. The following statements are equivalent:*

(i) *X is Kähler,*

(ii) *S does not contain any effective curve C which is a boundary, i.e. such that $[C] = 0$ in $H_2(X, \mathbb{Z})$.*

THEOREM II. — *Let X be a 1-convex manifold with 1-dimensional exceptional set S. If $H_2(X, \mathbb{Z})$ is finitely generated, then X is embeddable if and only if it is Kähler.*

The proofs do not depend on the results of M. Coltoiu and Vo Van Tan, nevertheless using them one can show that condition (ii) in Theorem I is equivalent to

(ii′) $\dim X \neq 3$ or S does not contain any effective curve which is a boundary and such that its irreducible components are rational curves of type $(-1, -1)$ or $(0, -2)$ or $(1, -3)$.

Theorems I and II give a partial answer to several questions of [16] (in particular to Question 2.6, Problem 2.7 and Problem 3.9), and support the idea of a parallelism between Moišezon and 1-convex manifolds (see the Introduction of [15]); more precisely, one should compare Theorem II with the fact that a Moišezon manifold is projective if and only if it is Kähler. Finally we remark that all the known examples of non-embeddable 1-convex manifolds with 1-dimensional exceptional set satisfy our hypothesis, that is $H_2(X, \mathbb{Z})$ is finitely generated. We also hope that our result will give some suggestion in order to study the general case.

Let us conclude with a remark on techniques; as to the author's knowledge, the duality between forms and currents on a non-compact manifold is used here for the first time (in the compact case, see [9], [10], [1] et al.), and this new tool provides quite elementary proofs.

Results.

First of all, we fix some notations and recall the characterization of 1-convex and embeddable manifolds which we need in what follows.

DEFINITION 1. — *Let X be a complex manifold.*

A real $(1,1)$-form $\varphi \in \mathcal{E}^{1,1}_{\mathbb{R}}(X)$ is called positive (semi-positive) at $x \in X$ if the hermitian form induced by φ_x on the holomorphic tangent space $T'_x X$ is positive definite (semi-definite); we shall write $\varphi_x > 0$ $(\varphi_x \geqslant 0)$. If $\varphi_x > 0$ $(\varphi_x \geqslant 0)$ for every $x \in X$, φ is called positive (semi-positive), and we shall write $\varphi > 0$ $(\varphi \geqslant 0)$.

A current $T \in \mathcal{E}^{1,1}_{\mathbb{R}}(X)'$ is positive (in the classical sense of Lelong) if $T(\varphi) \geqslant 0$, for every $\varphi \geqslant 0$.

An effective curve is a finite combination $\sum_{j=1}^{N} n_j C_j$ of irreducible curves C_j with integral coefficients $n_j > 0$.

Let L be a holomorphic line bundle on X endowed with a hermitian metic h; if Θ is the associated curvature form, we denote by $c_1(L,h) = \frac{i}{2\pi}\Theta$ its Chern form. L is positive if and only if there is a metric h such that $c_1(L,h) > 0$.

Let us denote by $\tilde{H}^p(X,\mathbb{Z})$ the range of the map $i^* : H^p(X,\mathbb{Z}) \to H^p(X,\mathbb{R})$ induced by the inclusion $i : \mathbb{Z} \to \mathbb{R}$. So a real, closed p-form ψ is called integral if its class $[\psi] \in H^p(X,\mathbb{R})$ belongs to $\tilde{H}^p(X,\mathbb{Z})$.

Consider the map

$$\delta : H^1(X, \mathcal{O}^*) \to H^2(X, \mathbb{Z}),$$

arising from the exact sequence

$$0 \to \mathbb{Z} \to \mathcal{O} \to \mathcal{O}^* \to 0.$$

It is well known that if L is a holomorphic line bundle, for every hermitian metric h on L, $c_1(L,h) \in i^*(\delta(L))$.

DEFINITION 2. — *A complex space X is said* 1*-convex (or strongly pseudoconvex) if it satisfies one of the following equivalent conditions:*

(i) *there exists a smooth exhaustion function $F : X \to \mathbb{R}$ which is strictly plurisubharmonic outside a compact subset of X,*

(ii) *there exists a proper surjective holomorphic map $f : X \to Y$ onto a Stein space Y, and a finite subset $B \subset Y$ such that, if $S := f^{-1}(B)$, the induced map $X \setminus S \to Y \setminus B$ is biholomorphic and $\mathcal{O}_Y \simeq f_*\mathcal{O}_X$.*

Remark 3. — The equivalence of the above conditions was proved in [11]. The map f is called the Remmert reduction, and S, which is the maximal compact analytic set of positive dimension in X, is called the exceptional set of X.

DEFINITION 4. — *A 1-convex space X is called embeddable if there is a holomorphic embedding $j : X \to \mathbb{C}^m \times \mathbb{CP}_n$, for some m and n.*

Roughly speaking, a 1-convex space X "differs" from a Stein space Y (its Remmert reduction) in its exceptional set S; Y is embeddable in some $\mathbb{C}^m$, and S is Moišezon and sometimes projective. Therefore it seems natural to ask when X is embeddable in $\mathbb{C}^m \times \mathbb{CP}_n$, for some m and n (this is not always the case, see [5]). As for the compact/projective case, to prove that a 1-convex space is embeddable, it is useful to search "positive" holomorphic line bundles on it: see [14] and [13]. Since we shall assume that X is a manifold, we use a simpler result, due to Eto, Kazama and Watanabe:

PROPOSITION 5 ([8], Theorem III). — *Let X be a 1-convex manifold carrying a positive holomorphic line bundle L. Then X is embeddable.*

To prove Theorems I and II, we need a couple of Lemmata; the first one is an elementary result in group theory.

LEMMA 6. — *If $g_1, \dots, g_q$ are elements of an abelian group G such that, for suitable real numbers $r_1, \dots, r_q > 0$, $g_1 \otimes r_1 + \dots + g_q \otimes r_q = 0$ in $G \otimes \mathbb{R}$, then there exist positive integers $n_1, \dots, n_q$ such that $n_1 g_1 + \dots + n_q g_q = 0$ in G.*

Proof. — Let G' be the subgroup of G generated by $g_1, \dots, g_q$. Since $G' \otimes \mathbb{R}$ is a subgroup of $G \otimes \mathbb{R}$, G' satisfies the hypotheses too; so there is no restriction in assuming that G is finitely generated. Therefore $G = F \oplus T$, where F is free and T is the torsion part, and $g_j = f_j + t_j, j = 1, \dots, q$, with $f_j \in F$ and $t_j \in T$. Thus by the hypothesis $f_1 \otimes r_1 + \dots + f_q \otimes r_q = 0$; identifying F with $\mathbb{Z}^n \subset \mathbb{R}^n$, for a suitable n, we get, in $\mathbb{R}^n$,

$$r_1 f_1 + \dots + r_q f_q = 0.$$

This turns out to be a linear system with integral coefficients: therefore there are positive integers $m_1, \dots, m_q$, such that $m_1 f_1 + \dots + m_q f_q = 0$. For a suitable positive integer m: $m t_1 = \dots = m t_q = 0$; thus

$$m m_1 g_1 + \dots + m m_q g_q = 0$$

Q.E.D.

LEMMA 7. — *Let $\varphi \in \mathcal{E}^{1,1}_{\mathbb{R}}(X)$ be a closed $(1,1)$-form on X which is positive on S (i.e. $\varphi_x > 0$, for every $x \in S$). Then there exists a smooth function $F : X \to \mathbb{R}$ such that $\varphi + \frac{i}{2\pi}\partial\overline{\partial}F$ is positive on X.*

Proof. — Let $g : X \to \mathbb{C}^N$ be the proper holomorphic map given by the composition of the Remmert reduction $f : X \to Y$ and an embedding $j : Y \to \mathbb{C}^N$. Denote by $z = (z_1, \dots, z_N)$ coordinates in $\mathbb{C}^N$ and let $\alpha := \frac{i}{2\pi}\partial\overline{\partial}|z|^2$; then $g^*\alpha \geqslant 0$ on X and $g^*\alpha > 0$ on $X \setminus S$. Since $\varphi > 0$ in a neighborhood of S , for every compact subset K of X there exists a positive constant M_K such that $\varphi + M_K g^*\alpha > 0$ on K.

Let $K_1 := \overline{B(1)} = \{z \in \mathbb{C}^N; |z| \leqslant 1\}$ and for every $n \geqslant 1$, $K_{n+1} := \overline{B(n+1)} \setminus B(n)$. Therefore, for suitable $M_n > 0$:

$$\varphi + M_n g^*\alpha > 0, \text{ on } g^{-1}(K_n).$$

Choose a smooth function $u : [0, +\infty) \to \mathbb{R}$ such that $u'(t) > 0$, for every $t \geqslant 0$, and $u(|z|^2) > M_n$ on K_n, for $n \geqslant 1$.

Let $v(s) := \int_0^s u(t)dt$, $s \geqslant 0$. We get

$$i\partial\overline{\partial}v(|z|^2) = v''(|z|^2)i\partial|z|^2 \wedge \overline{\partial}|z|^2 + v'(|z|^2)i\partial\overline{\partial}|z|^2.$$

The first term in the right hand side is $\geqslant 0$, hence on K_n we have

$$\frac{i}{2\pi}\,\partial\overline{\partial}v(|z|^2) > M_n\frac{i}{2\pi}\,\partial\overline{\partial}|z|^2;$$

therefore we get on X:

$$\varphi + g^*\frac{i}{2\pi}\,\partial\overline{\partial}v(|z|^2) > 0.$$

Q.E.D.

Let us go to the proofs of Theorem I and Theorem II.

THEOREM I. — *Let X be a 1-convex manifold with 1-dimensional exceptional set S. The following statements are equivalent:*

(i) *X is Kähler,*

(ii) *S does not contain any effective curve C which is a boundary, i.e. such that $[C] = 0$ in $H_2(X, \mathbb{Z})$.*

Proof. — (i) $\Rightarrow$ (ii): In a Kähler manifold no positive combination of compact analytic subsets can be a boundary.

(ii) $\Rightarrow$ (i): Let

$$A := \{\varphi \in \mathcal{E}^{1,1}_{\mathbb{R}}(X); \varphi_x > 0, \text{ for every } x \in S\}$$

and

$$B := \{\varphi \in \mathcal{E}^{1,1}_{\mathbb{R}}(X); d\varphi = 0\}.$$

Let us argue by contradiction and assume that $A \cap B = \emptyset$. Since A is a non-empty open cone in the natural topology of $\mathcal{E}^{1,1}_{\mathbb{R}}(X)$ and B is a linear subspace, by the Hahn-Banach Theorem ([12], II.3.1), we get a closed hyperplane in the topological vector space $\mathcal{E}^{1,1}_{\mathbb{R}}(X)$, containing B and not intersecting A. That is, there exists a current $T \in \mathcal{E}^{1,1}_{\mathbb{R}}(X)'$ such that $T(\varphi) > 0$ for every $\varphi \in A$, and $T(\varphi) = 0$ for every $\varphi \in B$.

CLAIM. — *Let* $T \in \mathcal{E}^{1,1}_{\mathbb{R}}(X)'$.

(i) *If* $T(\varphi) > 0$ *for every* $\varphi \in A$, *then* T *is positive and* $\operatorname{supp}(T) \subset S$;

(ii) *If* $T(\varphi) = 0$ *for every* $\varphi \in B$, *then* $i\partial\overline{\partial}T = 0$.

Proof of the claim. — Let $\psi \in \mathcal{E}^{1,1}_{\mathbb{R}}(X)$, $\psi \geqslant 0$, and let $\varphi \in A$: then for every $\varepsilon > 0$, $\psi + \varepsilon\varphi \in A$, hence

$$T(\psi) = \lim_{\varepsilon \to 0} T(\psi + \varepsilon\varphi) \geqslant 0.$$

Now let $\psi \in \mathcal{E}^{1,1}_{\mathbb{R}}(X)$, $\operatorname{supp}(\psi) \subset X \setminus S$, and let $\varphi \in A$. For every $c \in \mathbb{R}$, $c\psi + \varphi \in A$, so that

$$cT(\psi) + T(\varphi) > 0 \ \forall c \in \mathbb{R};$$

this implies $T(\psi) = 0$.

To prove the second assertion, notice that for every $f \in \mathcal{E}^{0}_{\mathbb{R}}(X)$, $i\partial\overline{\partial}f \in B$ so that $i\partial\overline{\partial}T(f) = T(i\partial\overline{\partial}f) = 0$. Q.E.D.

The claim allows to apply Theorem 4.10 in [3] to the current T arising from the Hahn-Banach Theorem: therefore, there are some irreducible components of S, say $C_1, \dots, C_q$, and some positive real numbers $r_1, \dots, r_q$ such that

$$T = \sum_{j=1}^{q} r_j C_j$$

(in particular, T is closed). Let $\varphi = \overline{\varphi}_{02} + \varphi_{11} + \varphi_{02}$ be a closed real 2-form on X; define $\pi(\varphi) := \varphi_{02}$. Then π induces a linear map $\Pi : H^2(X,\mathbb{R}) \to H^{0,2}_{\overline{\partial}}(X) = H^2(X,\mathcal{O})$. But S is 1-dimensional, thus ([4], Lemma 1) $H^2(X,\mathcal{O}) = 0$; since Π is the null map, every class belonging to $H^2(X,\mathbb{R})$ can be represented by means of an element of B. Hence "T vanishes on B" means that $\sum_{j=1}^{q} r_j[C_j]$ vanishes on every class of $H^2(X,\mathbb{R})$. But $H^2(X,\mathbb{R}) = \operatorname{Hom}(H_2(X,\mathbb{R}),\mathbb{R})$, thus $\sum_{j=1}^{q} r_j[C_j] = 0$ in

$H_2(X,\mathbb{R}) = H_2(X,\mathbb{Z}) \otimes \mathbb{R}$. Applying Lemma 6 we get a contradiction with (ii). Therefore there exists $\varphi \in A \cap B$; by means of Lemma 7 the form φ can be modified in order to obtain a closed Kähler form for X. Q.E.D.

Remark 8. — When S is irreducible, condition (ii) in Theorem I means that the class of S in $H_2(X,\mathbb{Z})$ is not a torsion class: therefore the map $H_2(S,\mathbb{R}) \to H_2(X,\mathbb{R})$ turns out to be injective and $\gamma : H^2(X,\mathbb{R}) \to H^2(S,\mathbb{R})$ is surjective. In this case there is another proof[1] of Theorem I. Denote by $\mathcal{H}$ the sheaf of germs of real pluriharmonic functions; as in [9], we get easily that $0 \to \mathbb{R} \to \mathcal{O} \to \mathcal{H} \to 0$ is exact and that $H^1(X,\mathcal{H})$ is the space of real closed $(1,1)$-forms, modulo the space of $i\partial\bar{\partial}$ -exact forms. Now for a suitable embeddable neighborhood U of S (see [6], p. 563) we get that in the following commutative diagram with exact lines:

$$\begin{array}{ccccccccc}
\dots & \to & H^1(X,\mathcal{O}) & \to & H^1(X,\mathcal{H}) & \to & H^2(X,\mathbb{R}) & \to & H^2(X,\mathcal{O})=0 \\
 & & \alpha\downarrow & & \beta\downarrow & & \gamma\downarrow & & \\
\dots & \to & H^1(U,\mathcal{O}) & \to & H^1(U,\mathcal{H}) & \to & H^2(U,\mathbb{R}) & \to & H^2(U,\mathcal{O})=0
\end{array}$$

α is an isomorphism, $H^2(U,\mathbb{R}) = H^2(S,\mathbb{R})$, γ is surjective. Hence β is surjective, thus we can extend a Kähler form on U to a closed $(1,1)$-form on X; by Lemma 7, X is Kähler.

THEOREM II. — *Let X be a 1-convex manifold with 1-dimensional exceptional set S. If $H_2(X,\mathbb{Z})$ is finitely generated, the following statements are equivalent:*

(i) *X is Kähler,*

(ii) *S does not contain any effective curve C which is a boundary, i.e. such that $[C]=0$ in $H_2(X,\mathbb{Z})$,*

(iii) *X is embeddable.*

Proof. — (iii) $\Rightarrow$ (i) is obvious, so by the previous theorem it is enough to show (ii) $\Rightarrow$ (iii). Let

$$B' := \{\varphi \in \mathcal{E}^{1,1}_{\mathbb{R}}(X); \varphi = c_1(L,h), \text{ for some holomorphic line bundle } L \text{ on } X \text{ and some hermitian metric } h \text{ on } L\}.$$

Let us assume $A \cap B' = \emptyset$. In order to argue exactly as in Theorem I, with B' instead of B, only two remarks are necessary. First, from $A \cap B' = \emptyset$ it follows $A \cap (B' \otimes \mathbb{R}) = \emptyset$ where $B' \otimes \mathbb{R}$ is the linear subspace generated by B'; so A and B' can be separated as above. Second, let f be a real smooth

[1] Private communication of M. Coltoiu.

function on X and L a holomorphic line bundle with a hermitian metric h; then

$$\frac{i}{2\pi}\partial\overline{\partial}f = c_1(L,h) - c_1(L,e^f h) \in B' \otimes \mathbb{R}.$$

Thus Claim (ii) in the proof of Theorem I holds with B' instead of B. Therefore there is a current $T = \sum_{j=1}^q r_j C_j$, with $r_j > 0$, which vanishes on B'.

As we have just noted, $H^2(X,\mathcal{O}) = 0$, thus the map $\delta : H^1(X,\mathcal{O}^*) \to H^2(X,\mathbb{Z})$ is surjective. Therefore each integral class can be represented by an element of B', thus $\sum_{j=1}^q r_j[C_j] \in H_2(X,\mathbb{R})$ vanishes on $\tilde{H}^2(X,\mathbb{Z})$. Since $H_2(X,\mathbb{Z})$ is finitely generated, $H^2(X,\mathbb{R}) = \tilde{H}^2(X,\mathbb{Z})\otimes\mathbb{R}$: thus $\sum_{j=1}^q r_j[C_j]$ vanishes on $H^2(X,\mathbb{R})$. As we have just seen in the previous proof, this gives a contradiction, so we conclude that there are L and h such that $c_1(L,h) \in A \cap B'$. Finally, applying Lemma 7, it follows that $c_1(L,e^{-F}h)$ is positive on X. Q.E.D.

COROLLARY 9. — *In the hypotheses of Theorem I, the statements (i), (ii) are also equivalent to*

(ii') $\dim X \neq 3$ or S does not contain any effective curve which is a boundary and such that its irreducible components are rational curves of type $(-1,-1)$ or $(0,-2)$ or $(1,-3)$.

Proof. — Obviously (ii) $\Rightarrow$ (ii'). Assume that (ii) is false, that is, there are positive integers $n_1,\dots,n_q$ and irreducible components $C_1,\dots,C_q$ of S such that $T = \sum_{j=1}^q n_j C_j$ is a boundary. Then the canonical bundle K_X vanishes on T, that is, $0 = \sum_{j=1}^q n_j K_X.C_j$, so $K_X.C_j = 0$ for every $j = 1,\dots,q$. By the Step 1 of Theorem 1.5 in [16], every C_j is the exceptional set of a 1-convex neighborhood U_j of it. Now $0 = K_X.C_j = K_{U_j}.C_j$; so K_{U_j} is not ample and, by means of Theorem 3 in [5], this implies that C_j is rational, $\dim X = 3$ and (see [5] , Remark p. 463) C_j must be of type $(-1,-1)$, $(0,-2)$ or $(1,-3)$. Q.E.D.

Remark 10. — All the examples given in the literature in connection to the embedding problem satisfy the property "$H_2(X,\mathbb{Z})$ is finitely generated"; nevertheless, using the example given in [7], it is easy to build a 1-convex threefold whose second homology group is not finitely generated.

Now assume that the exceptional curve S is irreducible. One can use the proof of Theorem II or apply the argument of Remark 8 to the sequence $0 \to \mathbb{Z} \to \mathcal{O} \to \mathcal{O}^* \to 0$, to see that (without the hypothesis

"$H_2(X,\mathbb{Z})$ is finitely generated") X is embeddable if and only if there exists a homomorphism in $\mathrm{Hom}(H_2(X,\mathbb{Z}),\mathbb{Z})$ which does not vanish on S, that is, the map $H^2(X,\mathbb{Z}) \to H^2(S,\mathbb{Z})$ is not the zero map.

Therefore a comparison beetwen this last condition and Theorem I may suggest the way to a counterexample (if it exists!) of a 1-convex Kähler manifold, with 1-dimensional exceptional set, which is non-embeddable.

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