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A HAMILTONIAN VERSION OF A RESULT OF GROMOLL AND GROVE

by Urs FRAUENFELDER, Christian LANGE & Stefan SUHR (*)

Abstract. — The theorem that if all geodesics of a Riemannian two-sphere are closed they are also simple closed is generalized to real Hamiltonian structures on $\mathbb{R}P^3$. For reversible Finsler 2-spheres all of whose geodesics are closed this implies that the lengths of all geodesics coincide.

Résumé. — On généralise aux structures hamiltoniennes réelles sur $\mathbb{R}P^3$ le théorème qui dit que, dans une 2-sphère riemannienne dont les géodésiques sont toutes fermées, toute géodésique est simplement fermée. Cela implique que, dans une 2-sphère finslerienne réversible dont les géodésiques sont toutes fermées, elles ont toutes la même longueur.

1. Introduction

In [10] Poincaré advertised the study of geodesics as a first step in the exploration of Hamiltonian systems like the ones appearing in celestial mechanics. A fascinating object in the study of geodesics are manifolds all of whose geodesics are closed, see [1]. This leads to the question which results on closed geodesics are Riemannian phenomena or belong to the realm of Hamiltonian dynamics. For example in [5] it was shown how the classical result of Bott and Samelson fits into a contact geometric set-up.

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The Theorem of Gromoll and Grove [7] asserts that if a Riemannian metric on the two-dimensional sphere S^2 has the property that all geodesics are closed, then each geodesic is simple closed and therefore all geodesics have a *common* minimal period.

The geodesic equation is a second order ODE. Each second order ODE can be transformed into a first order ODE. This allows one to interpret the geodesic equation as a first order Hamiltonian flow equation on the unit cotangent bundle of the underlying manifold. In the case of S^2 the unit cotangent bundle S^*S^2 is diffeomorphic to the real projective space $\mathbb{R}P^3$. In physics the underlying manifold is referred to as the configuration space while its cotangent space is denoted as the phase space.

The phase space is a symplectic manifold. The symplectic geometer is interested in applying arbitrary symplectic transformations to this symplectic manifold since under these transformations the symplectic dynamics transforms canonically and can sometimes be brought to a form which makes it more tractible to explicit solutions. But under arbitrary symplectic transformations the bundle structure of the cotangent bundle is not preserved. Therefore the question if a closed characteristic of the Hamiltonian flow is simple or not does not make sense to the symplectic geometer. However, in the case that all characteristics are closed, the question if the Hamiltonian flow admits a *common* minimal period, can even be understood by a symplectic geometer.

Unfortunately, in general this is bloody wrong. Here is an example. Let p and q be two relatively prime integers. Identify the three-dimensional sphere S^3 with $\{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 = 1\}$ and consider the circle action on S^3 given by

$$e^{it}_*(z_1, z_2) = (e^{ipt} z_1, e^{iqt} z_2).$$

This circle action commutes with the antipodal involution $z \mapsto -z$ on S^3 and hence induces a circle action on $\mathbb{R}P^3 = S^3/\mathbb{Z}_2$. The minimal period of the orbit through $[(1,0)]$ is $\frac{\pi}{p}$, the minimal period of the orbit through $[(0,1)]$ is $\frac{\pi}{q}$, while the minimal period of an orbit through a point $[(z_1, z_2)] \in \mathbb{R}P^3$ with $z_1 \neq 0$ and $z_2 \neq 0$ is π in the case p and q are both odd and 2π if one of them is even. It is worth noting that this example corresponds to the Katok examples as was pointed out by Harris and Paternain [8]. This means that the Theorem of Gromoll and Grove already fails for nonreversible Finsler metrics.

Geodesic flows of a Riemannian or a reversible Finsler metric have the following properties which distinguish them from other Hamiltonian flows:

(1) The geodesic flow is invariant under time reversal, i.e., a geodesic traversed backwards is again a geodesic.
(2) The geodesic flow is a Reeb flow, i.e., it is the flow of a Reeb vector field of a contact structure.

In this note we explain how under a suitable generalization of property (1) for geodesics the Theorem of Gromoll and Grove about the common minimal period generalizes to hold for general Hamiltonian systems all of whose trajectories are closed on energy hypersurfaces having the same topology as the ones studied by Gromoll and Grove, namely $\mathbb{R}P^3$. Here we do not need to assume that our Hamiltonian flow is a Reeb flow. Instead, we explain, invoking a deep result of Epstein [4], that in this set-up the Hamiltonian flow will be stable.

The proof of Gromoll and Grove hinges on the Theorem of Lusternik and Schnirelmann on the existence of three simple closed geodesics and the theory of Seifert on three-dimensional fibred spaces. As explained above there is no analogue of the Theorem of Lusternik and Schnirelmann for Hamiltonian flows because the notion of simpleness does not generalize to the symplectic set-up. However, the tangent curve of a smooth regular simple curve (i.e. an embedding) $S^1 \to S^2$ is a noncontractible curve in $TS^2 \setminus S^2$. We will show that the noncontractibility of all orbits follows without further assumptions. The analogue in the Riemannian case is proven in [9]. Here we employ the same method.

2. Real Hamiltonian manifolds

Assume that $\Sigma = \Sigma^{2n+1}$ is a closed connected oriented manifold of dimension $2n+1$. A *Hamiltonian structure* on Σ is a two form

$$\omega \in \Omega^2(\Sigma)$$

satisfying the following two conditions:

(1) ω is closed, i.e., $d\omega = 0$,
(2) $\ker(\omega)$ defines a one-dimensional distribution on $T\Sigma$.

We refer to the pair (Σ, ω) as a *Hamiltonian manifold*. A Hamiltonian manifold is the odd dimensional analogon of a symplectic manifold. In the study of Hamiltonian dynamics Hamiltonian manifolds naturally arise as energy hypersurfaces, i.e., level sets of a smooth function on a symplectic manifold. The characteristic foliation, namely the leaves of the kernel of ω, correspond to the trajectories of the Hamiltonian flow on a fixed energy level. People studying Hamiltonian manifolds are relaxed people: They do not care about the parametrization of the leaves but only about the foliation itself.

Because we assume our manifold Σ to be oriented we actually obtain an orientation of the leaves. Indeed, note that for each $x \in \Sigma$ the two form ω_x is nondegenerate on $T_x\Sigma/\ker(\omega_x)$, hence is a symplectic form on this vector space and therefore defines an orientation on it.

Definition 2.1. — *A* real Hamiltonian manifold *is a triple* (Σ, ω, ρ), *where* (Σ, ω) *is a Hamiltonian manifold and* $\rho \in \mathrm{Diff}(\Sigma)$ *is a smooth involution on* Σ, *i.e.,* $\rho^2 = \mathrm{id}_\Sigma$ *satisfying the following two conditions:*

(1) $\rho^*\omega = -\omega$,
(2) ρ *reverses the orientation on* $\ker(\omega)$.

If $\mathscr{L}$ denotes the set of leaves of the foliation on Σ induced from $\ker(\omega)$, then the first condition guarantees that ρ induces an involution on the set of leaves

$$\rho_* \colon \mathscr{L} \to \mathscr{L}.$$

Here is an example of a real Hamiltonian manifold. For a closed Riemannian manifold (N, g) let

$$\Sigma = S^*N = \{(q,p) \in T^*N : \|p\|_{g_q} = 1\}$$

be its unit cotangent bundle with Hamiltonian structure obtained by restricting the differential of the Liouville one-from to S^*N. Then a real structure is given by

$$\rho \colon S^*N \to S^*N, \quad (q,p) \mapsto (q,-p).$$

Note that the same construction works for reversible Finsler metrics but fails for nonreversible Finsler metrics since in this case the unit cotangent bundle is not invariant under ρ anymore. In this example the leaves of the foliation correspond to oriented geodesics and the involution reverses the orientation of the geodesic. In particular, no leaf is fixed. This is a more general phenomenon as the following lemma shows.

Lemma 2.2. — *Suppose that* (Σ, ω, ρ) *is a real Hamiltonian manifold with the property that*

$$\mathrm{Fix}(\rho) = \emptyset,$$

i.e., there are no fixed points of the involution ρ *on* Σ. *Then the induced involution* ρ_* *on the set of leaves* $\mathscr{L}$ *is free.*

Proof. — We argue by contradiction and assume that there exists a leaf $L \in \mathscr{L}$ such that

$$\rho_*(L) = L.$$

Hence if $x \in L$ it holds that

$$\rho(x) \in L.$$

Because ρ has no fixed points by assumption we must have

$$x \neq \rho(x).$$

The leaf L is either diffeomorphic to a circle or to the real line. Hence we can choose

$$I \subset L$$

diffeomorphic to a closed interval such that

$$\partial I = \{x, \rho(x)\}.$$

Note that since ρ is an involution it interchanges the boundary points of I. Because ρ reverses the orientation of L we conclude that

$$\rho(I) = I.$$

However, because ρ interchanges the boundary points of the closed interval I we conclude by the intermediate value theorem that there exists $y \in I$ such that

$$\rho(y) = y$$

contradicting the assumption that $\mathrm{Fix}(\rho) = \emptyset$. This contradiction concludes the proof of the lemma. □

3. Stability

Assume that (Σ, ω) is a Hamiltonian manifold of dimension $2n+1$. A one-form $\lambda \in \Omega^1(\Sigma)$ is called a *stabilizing one-form*, see [2, 3], if it satisfies the following two conditions

(1) $\lambda \wedge \omega^n > 0$, i.e., $\lambda \wedge \omega^n$ is a volume form on the oriented manifold Σ,
(2) $\ker(\omega) \subset \ker(\mathrm{d}\lambda)$.

We abbreviate by

$$\Lambda(\Sigma, \omega) \subset \Omega^1(\Sigma)$$

the maybe empty subset of all stabilizing one-forms of (Σ, ω). Note that $\Lambda(\Sigma, \omega)$ is a convex cone, i.e., if $\lambda_1, \lambda_2 \in \Lambda(\Sigma, \omega)$ and $r_1, r_2 > 0$, then $r_1\lambda_1 + r_2\lambda_2 \in \Lambda(\Sigma, \omega)$. We refer to $\Lambda(\Sigma, \omega)$ as the *stable cone* of (Σ, ω).

Definition 3.1. — *A Hamiltonian manifold* (Σ, ω) *is called* stable*, if* $\Lambda(\Sigma, \omega) \neq \emptyset$*, i.e., there exists a stabilizing one-form.*

Now suppose that (Σ, ω, ρ) is a real Hamiltonian manifold. We say that a stabilizing one-form $\lambda \in \Lambda(\Sigma, \omega)$ is *real* if it satisfies

$$\rho^*\lambda = -\lambda.$$

To produce real stabilizing one-forms we first show that the cone $\Lambda(\Sigma, \omega)$ is invariant under the involution $\lambda \mapsto -\rho^*\lambda$ of $\Omega^1(\Sigma)$.

Lemma 3.2. — *Suppose that* $\lambda \in \Lambda(\Sigma, \omega)$*. Then*

$$-\rho^*\lambda \in \Lambda(\Sigma, \omega).$$

Proof. — We know that $\ker \omega$ is oriented by the assumptions that Σ is oriented and that ω is a Hamiltonian structure. The involution ρ switches the orientation of $\ker \omega$, so the sign of $\rho^*\lambda|_{\ker \omega}$ is the opposite of $\lambda|_{\ker \omega}$. From $\rho^*\omega = -\omega$ we conclude

$$(-1)^{n+1}\rho^*(\lambda \wedge \omega^n) > 0.$$

Thus we have

$$(-\rho^*\lambda) \wedge \omega^n = (-1)^{n+1}\rho^*(\lambda \wedge \omega^n) > 0,$$

and the first condition for a stablizing one-form is satisfied by $-\rho^*\lambda$.

Combining $\ker(\rho^*\omega) = \ker \omega$ with $\ker \omega \subset \ker \mathrm{d}\lambda$ implies

$$\ker \omega = \ker \rho^*\omega \subset \ker \rho^*\mathrm{d}\lambda = \ker \mathrm{d}(-\rho^*\lambda).$$

Therefore $-\rho^*\lambda$ is a stabilizing one-form. □

In view of the convex cone property of $\Lambda(\Sigma, \omega)$ the lemma enables us to construct real stabilizing one-forms out of a stabilizing one-form.

Corollary 3.3. — *Suppose that the real Hamiltonian manifold* (Σ,ω,ρ) *is stable. Then it admits a real stabilizing one-form.*

Proof. — Let λ be a stabilizing one-form. By Lemma 3.2 it follows that $-\rho^*\lambda$ is again a stabilizing one-form. Because $\Lambda(\Sigma, \omega)$ is a convex cone it follows that

$$\lambda_\rho := \lambda - \rho^*\lambda$$

is again a stabilizing one-form. Because ρ is an involution we have

$$\rho^*\lambda_\rho = -\lambda_\rho.$$

Therefore λ_ρ is a real stabilizing one-form. □

4. Periodic Hamiltonian manifolds

Definition 4.1. — *A Hamiltonian manifold* (Σ, ω) *is called* periodic *if all leaves are circles.*

Theorem 4.2. — *Assume that* (Σ, ω) *is a* 3*-dimensional periodic Hamiltonian manifold. Then it is stable.*

Proof. — For every Hamiltonian structure on an orientable manifold one can orient the characteristic distribution. Therefore by choosing a nowhere vanishing vector field tangent to the characteristic distribution one finds that the characteristic foliation consists of orbits of a $\mathbb{R}$-action on Σ.

According to the main theorem in [4] if (Σ, ω) is periodic on the 3-manifold Σ then the $\mathbb{R}$-action is orbit equivalent to an S^1-action. We choose a Riemannian metric g with respect to which this S^1-action is isometric. Let X be a unit vector field with respect to g on Σ generating $\ker \omega$. Then by the Theorem of Wadsley in the version of [3, Theorem 2.3] the form

$$\lambda := i_X g$$

is a stabilizing one-form. □

Corollary 4.3. — *Under the assumptions of Theorem 4.2 there exists a smooth circle action on* Σ *without fixed points such that the orbits of the circle action correspond to the leaves of the characteristic foliation of* (Σ, ω).

5. The main result and proof

Theorem 5.1. — *Suppose that* (ω, ρ) *is a periodic real Hamiltonian structure on* $\mathbb{R}P^3$ *with* $\mathrm{Fix}(\rho) = \emptyset$*. Then there exists a free circle action on* $\mathbb{R}P^3$ *with the property that the leaves of* $(\mathbb{R}P^3, \omega)$ *correspond to the orbits of the circle action. In particular, all leaves are noncontractible.*

Corollary 5.2. — *Let* (S^2, F) *be a reversible Finsler* 2*-sphere such that all geodesics are closed. Then all geodesics give rise to noncontractible orbits on the unit tangent bundle of* S^2 *and have the same length.*

Remark 5.3. — If, in the situation of the corollary, there exists at least one simple closed geodesic, then a connectedness argument shows that in fact all geodesics are simple closed (cf. [7]). The existence of such a simple closed geodesic on any Finsler 2-sphere is not known in general but expected to hold.

In his fundamental monograph [12] Seifert studies effective circle actions on closed three-dimensional manifolds without fixed points. For a modern account of Seifert's theory see for example the paper by Scott [11]. If the three-dimensional manifold is orientable the quotient orbifold is an orientable surface with finitely many cone points. For a surface S and a finite collection of positive integers $p_1, \dots, p_\ell$ bigger than two we denote the surface with ℓ cone points of order $p_1, \dots, p_\ell$ by $S(p_1, \dots, p_\ell)$.

THEOREM 5.4 (Seifert). — *Suppose that S^1 acts smoothly on $\mathbb{R}P^3$ without fixed points. Then $\mathbb{R}P^3/S^1$ is an orbifold of one of the following types*

$$S^2, \quad S^2(p), \quad S^2(p,q).$$

In the third case it holds that

$$\gcd(p,q) \in \{1,2\},$$

i.e., p and q are either relatively prime or decompose as $p = 2p'$ and $q = 2q'$ with p' and q' relatively prime.

Proof of Theorem 5.4. — For the fibration $S^1 \to \mathbb{R}P^3 \to \mathbb{R}P^3/S^1$ we obtain an exact sequence of homotopy groups

$$\pi_1(S^1) \to \pi_1(\mathbb{R}P^3) \to \pi_1^{\mathrm{orb}}(\mathbb{R}P^3/S^1) \to \pi_0(S^1)$$

see [11, Lemma 3.2]. Hence we have an exact sequence

$$\mathbb{Z} \to \mathbb{Z}_2 \to \pi_1^{\mathrm{orb}}(\mathbb{R}P^3/S^1) \to \{1\}$$

implying that the fundamental group of the orbifold $\mathbb{R}P^3/S^1$ is either trivial or $\mathbb{Z}_2$. Closed two-dimensional orbifolds of trivial fundamental group or fundamental group $\mathbb{Z}_2$ are the three examples in the list above as we explain in the appendix. This finishes the proof of the theorem. □

COROLLARY 5.5. — *Suppose that (ω, ρ) is a real Hamiltonian structure on $\mathbb{R}P^3$ with the property that $\mathrm{Fix}(\rho) = \emptyset$. Then the space of leaves $\mathscr{L}$ is diffeomorphic to S^2 and all orbits are noncontractible.*

Theorem 5.1 follows directly from the corollary.

Proof of Corollary 5.5. — By Corollary 4.3 there exists a circle action on $\mathbb{R}P^3$ without fixed points such that the space of leaves is

$$\mathscr{L} = \mathbb{R}P^3/S^1.$$

Because the involution ρ has no fixed points by assumption, Lemma 2.2 tells us that the induced involution ρ_* on the space of leaves $\mathscr{L}$ is free. In particular, the number of cone points of a given multiplicity is even.

Therefore by Theorem 5.4 the space of leaves belongs to one of the following orbifolds

$$S^2, \quad S^2(p,p)$$

where in the later case we have

$$\gcd(p,p) \in \{1,2\}$$

implying that

$$p = 2.$$

By [6, Example 4.18] there does not exist an effective S^1-action without fixed points on the lens space $L(2,1)$ with orbit space $S^2(2,2)$ (cf. [9, Lemma 3.6]). Here the lens space $L(r,1)$ is defined as $S^3/\mathbb{Z}_r$ where the $\mathbb{Z}_r$-action is generated by multiplication with $\mathrm{e}^{\frac{2\pi}{r}\mathrm{i}}$. This excludes the orbifold $S^2(2,2)$ as the orbit space since $\mathbb{R}P^3 \cong L(2,1)$. Using the homotopy exact sequence

$$\pi_1(S^1) \to \pi_1(\mathbb{R}P^3) \to \pi_1(S^2) = 0$$

we see that all orbits are noncontractible. This finishes the proof of the corollary. □

Appendix. The orbifold fundamental group

Assume that Σ is an orientable closed 3-dimensional manifold on which the circle S^1 acts effectively and without fixed points. Then the quotient $S := \Sigma/S^1$ is an orbifold, namely a closed orientable surface with finitely many cone points (say ℓ many) of multiplicity p_i for $1 \leqslant i \leqslant \ell$. Suppose that the genus of the surface S is g. Its fundamental group as a topological space has the presentation

$$\pi_1(S) = \left\langle a_1, b_1, \ldots, a_g, b_g \;\middle|\; a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1} \right\rangle .$$

Its *orbifold fundamental group* is defined as

$$\pi_1^{\mathrm{orb}}(S) = \langle a_1, b_1, \ldots, a_g, b_g, x_1, \ldots, x_\ell \\ \left| \; x_1^{p_1}, \ldots, x_\ell^{p_\ell}, a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1} x_1 \cdots x_p \right\rangle .$$

The usefulness of the orbifold fundamental group for us lies in the fact that one has a surjective group homomorphism

$$\pi_1(\Sigma) \twoheadrightarrow \pi_1^{\mathrm{orb}}(S)$$

as was proved by Seifert in [12, Chapter 10]. This comes from the homotopy exact sequence

$$\pi_1(S^1) \to \pi_1(\Sigma) \to \pi_1^{\mathrm{orb}}(S) \to \pi_0(S^1),$$

see for example [11, Chapter 3]. The existence of this surjective group homomorphism gives us interesting information on the types of exceptional fibers the action of S^1 on Σ can have.

Suppose in the following that the fundamental group of Σ is finite. In view of the surjective group homomorphism the orbifold fundamental group of S is finite as well. In particular, topologically S has to be a sphere and its orbifold fundamental group has the presentation

$$\pi_1^{\mathrm{orb}}(S) = \langle x_1, \dots, x_\ell \mid x_1^{p_1}, \dots, x_\ell^{p_\ell}, x_1 \cdots x_p \rangle .$$

It turns out that this group is finite only if there are either less than three cone points or three cone points and the orbifold is of the form

$$S^2(2,2,n),\ S^2(2,3,3),\ S^2(2,3,4) \text{ or } S^2(2,3,5)$$

for $n \geqslant 2$. For a beautiful geometric interpretation why this holds true we invite the reader to consult directly Chapter 10 in Seifert's paper [12]. In the case where there is just one cone point, the orbifold is a teardrop, and its fundamental group is trivial. If there are two cone points the fundamental group is

$$\pi_1^{\mathrm{orb}}(S^2(p,q)) \cong \mathbb{Z}_k$$

where $k = \gcd(p,q)$ is the greatest common divisor of the integers p and q. The fundamental group of $S^2(2,2,n)$ is the dihedral group of order $2n$ while the fundamental groups $S^2(2,3,3)$, $S^2(2,3,4)$ and $S^2(2,3,5)$ correspond to the symmetry groups of the platonic solids. In particular, we see that if the orbifold fundamental group of S is trivial or $\mathbb{Z}_2$, then the orbifold belongs to one of the examples listed in Theorem 5.4.

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